

Changes in the Financial Market: Welfare and Price Effects and the Basic Theorems of Value Conservation

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ABSTRACT

This paper analyzes the impact, on both welfare and equilibrium prices, of changes in the financial market in a general equilibrium, two-period context. Previous papers have focussed on the "securities effect," tending to essentially ignore the equally important "endowment effect" that arises when market structure changes are implemented. Two forms of endowment neutrality and market structure changes which either preserve, expand, or shift allocational feasibility differentiate the main theorems, which are based on arbitrary preferences and beliefs and substantially extend and modify extant results; in particular, earlier statements identified with value conservation are sharply moderated. Very roughly, the paper yields the following implications for some of the more common changes in the market: nonsynergistic corporate spinoffs and the opening of option markets have, on balance, strongly positive welfare effects; nonsynergistic mergers tend to have strong negative welfare effects, while the welfare effects of alternative risky debt structures tend to be ambiguous. All of the preceding, however, may under plausible conditions be redistributive.

CHANGES IN THE STRUCTURE of the financial market have long been of interest to financial economists. While such changes may take many forms, it is perhaps surprising that only a few of the more common ones have been systematically studied. Foremost among these are changes which involve the firm's capital structure (the relative amounts of debt and equity), mergers, and other special recapitalizations.

This paper analyzes the impact, on both welfare and equilibrium prices, of changes in the financial market in a two-period context. It differs from previous studies primarily in that it is based on a fully integrated (general equilibrium) approach similar to that employed in international trade analysis. Thus, while the resulting mosaic contains previous studies as clearly recognizable fragments, such as the classic paper of Modigliani and Miller [33], it also provides the necessary framework and tools for an evaluation of market structure changes of *any* type, such as changes involving subordinated debt, convertibles, warrants, mergers, spinoffs, and the opening of option markets.

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Three key features characterize the approach used. First, market structure changes are grouped into three types: those which preserve the space of feasible allocations (Type I); those which expand (or contract) it (Type II); and those which cause nontrivial shifts in the set of feasible allocations (Type III). Second, the focus of attention is on "global" statements, i.e. statements which can be made independently of preferences and beliefs. Previous studies, in contrast, have concentrated almost exclusively on specialized preference and belief structures (e.g., Mossin [35], Rubinstein [41]), and on Type I cases. Finally, endowment effects are found to play a crucial role in the analysis; the broader ramifications of such effects have, with few exceptions, notably Litzenberger and Sosin [27] and Sosin [48], previously been ignored or overlooked. When taken into account, they tend to sharply moderate earlier statements identified with value conservation, for example. Thus, when the set of securities in the market is altered, there are *two* separable and equally important effects: one resulting from the change in feasible portfolio choices and one from the (generally unavoidable) substitution of endowment patterns. Endowment preservation or neutrality may take two forms: weak or strong. Weak endowment neutrality holds when both markets have the same equilibrium implicit price structure and the equilibrium values of the endowments in both markets are the same. Strong endowment neutrality is even more demanding: it requires that everyone's state by state payoff from the endowment securities given up be preserved by the (endowment) securities received in their place.

In a larger sense, the present paper may be viewed as an attempt to shed further light on the demand for financial assets, a subject which is yet to be integrated with the supply side of financial markets. While the supply conditions have not received nearly as much attention as the demand conditions, a promising beginning has been made by Jensen and Meckling [24] who examine how "agency costs" affect the issuance of financial securities. In any event, it is in the context of eventual integration with costly supply conditions that (demand) comparisons between (perfect) markets which achieve less than full allocational efficiency assume particular relevance.

The results of the paper may be sketched as follows: for arbitrary beliefs, preferences, and endowment shifts, the following "weak" results obtain (Theorems 1, 5, 8): only feasibility preserving market structure changes preclude Pareto-dominance of one market structure over another; only feasibility expanding changes rule out the Pareto inferiority of the new market, while feasibility shifting changes, not surprisingly, have unrestricted welfare effects. In general, price effects and welfare effects appear to be essentially independent of each other, so that wealth, for example, may decrease as everyone becomes better off.

Contrary to common belief, feasibility preserving changes in the market do *not* imply value conservation and investor indifference but may be redistributive, increasing welfare for some and decreasing it for others. The attainment of investor indifference and value conservation requires the imposition of much stronger conditions; there must also be weak endowment neutrality accompanied by unique equilibria (in both markets) *or* there must be strong endowment neutrality plus endowments in the first market that constitute an efficient allocation in that market (Theorems 3, 4, 3', and 4'). Returning to the redistri-

butive possibilities, only when strong endowment neutrality (which is often unattainable) holds is each equilibrium in Market 1 also an equilibrium in Market 2 (Theorem 2). In consequence, value and welfare conservation should, even in feasibility preserving market structure changes, probably be viewed as the exception rather than the rule.

As far as I am aware, feasibility expanding market structure changes were first examined by Borch [6, pp. 95–103] but have not previously been subjected to systematic analysis. While, as noted, Pareto inferior allocations are always precluded, redistributive effects are certain to be avoided only if either endowments in Market 1 constitute an efficient allocation in that market *and* strong endowment neutrality holds (Theorem 6), or the two markets yield unique equilibria, have a common set of implicit prices, and weak endowment neutrality holds (Theorem 7). Finally, in moving to a more constrained market structure (Type II reversed), Corollary 3 confirms that (values and) welfare can be conserved only under very specialized conditions.

Very roughly, the preceding yields the following implications for some of the more common changes in the market: nonsynergistic corporate spinoffs and the opening of option markets have, on balance, strongly positive welfare effects; nonsynergistic mergers tend to have strongly negative welfare effects, while the welfare effects of alternative risky debt structures tend to be ambiguous. All of them, however, may under certain conditions be redistributive.

The paper proceeds as follows. The basic model and its equilibrium properties are specified in Section I. Section II formalizes the relationship between market structures and feasible allocations. Section III defines endowment neutrality, which, as noted, plays a crucial role in the analysis. Several of the basic issues which arise in comparing equilibria are analyzed in Section IV, which also contains some of the underpinning results. The main theorems on weak and strong market equivalence are given in Section V. Feasibility expanding (reducing) market structure changes are addressed in Section VI, while feasibility shifting changes are examined in Section VII. Section VIII depicts some of the results in graphic form, while Section IX illustrates the tenuous relationship between prices and welfare. The conditions which imply value conservation are summarized in Section X. Section XI applies the findings of the paper to corporate spinoffs, mergers, option markets, and a sprinkling of corporate capital structure decisions while Section XII contains some concluding comments.

I. Preliminaries

We consider a pure exchange economy with a single commodity which lasts for two periods under the standard assumptions.¹ That is, at the end of Period 1 the economy will be in some state s , where $s = 1, \dots, n$. There are \bar{I} consumer-investors indexed by i , whose probability beliefs over the states are given by the

¹ Most previous studies have assumed a single commodity in a one- or two-period setting. It should be noted, however, that the introduction of multiple commodities or more than two periods is a nontrivial step which may bring about additional complications, such as Pareto-dominated equilibria, as shown by Hart [20].

vectors $\pi_i = (\pi_{i1}, \dots, \pi_{in})$, where

$$\pi_{is} > 0, \quad \text{all } i, s \quad (1)$$

Thus, while probability assessments are permitted to differ among investors, none assigns probability 0 to any of the identified states² 1, ..., n . The preferences of consumer-investor i are represented by the (conditional) functions $U_{is}(c_i, w_{is})$, where c_i is the consumption level in Period 1, and w_{is} is the consumption level in Period 2 if the economy is in state s at the beginning of that period. These functions are defined for

$$c_i \geq 0 \quad \text{and} \quad w_{is} \geq 0 \quad (2)$$

and are assumed to be increasing and strictly concave.

At the beginning of Period 1 (time 0), consumer-investors allocate their resources among current consumption c_i and a portfolio chosen from a set J of securities indexed by j . Security j pays $a_{js} \geq 0$ per share at the end of Period 1 and the total number of outstanding shares is Z_j . Let z_{ij} denote the number of shares of security j purchased by investor i at time 0; his portfolio $z_i = (z_{i1}, \dots, z_{iJ})$ then yields the payoff

$$w_{is} = \sum_{j \in J} z_{ij} a_{js}$$

available for consumption in Period 2, if state s occurs at the end of Period 1. Investor endowments are denoted (\bar{c}_i, \bar{z}_i) and financial markets, as is usual, are assumed to be competitive and perfect.³ The number of securities, however, need not be large (although this is not ruled out). If the rank of matrix $A = [a_{js}]$ is full (equals n), the financial market will be called complete; if not, it will be called incomplete.

Aggregate wealth or consumption in state s is given by

$$W_s \equiv \sum_{j \in J} Z_j a_{js}, \quad \text{all } s$$

and is assumed to be positive and bounded. Finally, we also assume that everyone's endowment (\bar{c}_i, \bar{z}_i) is such that it satisfies

$$(\bar{c}_i, \bar{w}_i) \neq 0 \quad \text{and} \quad \bar{c}_i \geq 0, \quad \bar{w}_{is} \geq 0, \quad \text{all } i, s \quad (3)$$

Condition (3) insures that each consumer-investor faces a feasible and nontrivial decision problem.

Under our assumptions, each consumer-investor i maximizes

$$u_i \equiv \sum_s \pi_{is} U_{is}(c_i, \sum_{j \in J} z_{ij} a_{js}) \quad (4)$$

with respect to the decision vector (c_i, z_i) , subject to (2) and to his budget

² Assumption (1) can be slightly relaxed without jeopardizing the existence of equilibrium (see below). However, by employing (1), we avoid confronting the possibility that state s actually occurred even though some individuals assessed π_{is} to be 0—which, in turn, raises issues beyond the scope of this paper.

³ That is, consumer-investors perceive prices as beyond their influence, there are no transaction costs or taxes, securities and commodities are perfectly divisible, and the full proceeds from short sales can be invested.

constraint

$$c_i P_0 + \sum_{j \in J} z_{ij} P_j = \bar{c}_i P_0 + \sum_{j \in J} \bar{z}_{ij} P_j$$

as a price-taker, where P_0 is the price of a unit of Period 1 consumption and P_j is the price of security j . In equilibrium, prices will be such that all markets clear.

In view of assumptions (1)–(3), an equilibrium will exist but need not be unique⁴ (see e.g. Hart [19] and Milne [31, 32]). The equilibrium conditions for any market structure A may be written

$$\sum_s \pi_{is} \frac{\partial U_{is}(c_i, \sum_{j \in J} z_{ij} a_{js})}{\partial c_i} + \delta_{i0} = \lambda_i \quad \text{all } i \quad (5)$$

$$\sum_s \pi_{is} \frac{\partial U_{is}(c_i, \sum_{j \in J} z_{ij} a_{js}) a_{js}}{\partial w_{is}} + \sum_s \delta_{is} a_{js} = \lambda_i P_j \quad \text{all } i, j \quad (6)$$

$$c_i \geq 0, \quad z_i A \geq 0 \quad \text{all } i \quad (7)$$

$$c_i + z_i P = \bar{c}_i + \bar{z}_i P \quad \text{all } i \quad (8)$$

$$\delta_{i0} \geq 0, \quad \delta_{is} \geq 0, \quad \delta_{i0} c_i = 0, \quad \delta_{is} w_{is} = 0 \quad \text{all } i, s \quad (9)$$

$$\sum_i c_i = \sum_i \bar{c}_i, \quad \sum_i z_{ij} = Z_j \quad \text{all } j \quad (10)$$

where the λ_i and δ_{is} are Lagrange multipliers, (10) represents the market clearing equations, and P_0 has been chosen as numeraire, i.e. $P_0 \equiv 1$. In the system (5)–(10), $\delta_{is} = 0$ corresponds to an allocation which is interior with respect to the relevant non-negativity constraint on consumption.

We note that an allocation (c^*, z^*) which constitutes a solution to system (5)–(10) (along with a price vector P , a vector λ , and a matrix δ) is sensitive to (1) the set of preferences; (2) the set of beliefs; (3) *the distribution of endowments*; and (4) the market structure A . In addition, (c^*, z^*) is *allocationally efficient with respect to the market structure A*. However, there may exist trades *outside* the market (e.g. by the invention of new securities) which yield allocations that Pareto-dominate the market allocation (c^*, z^*) . When this is not the case, i.e. when (c^*, z^*) is allocationally efficient with respect to *all* conceivable allocations inside and outside the existing market, (c^*, z^*) will be said to be *fully allocationally efficient*.

To be more precise, let

$$P_{is} \equiv \frac{1}{\lambda_i} \left\{ \pi_{is} \frac{\partial U_{is}(c_i^*, \sum_{j \in J} z_{ij}^* a_{js})}{\partial w_{is}} + \delta_{is} \right\}$$

It is well known that (5)–(10) plus

$$P_{is} = P_{1s} \quad \text{all } i \geq 2, \quad \text{all } s \quad (11)$$

is a necessary and sufficient condition for the market allocation (c^*, z^*) to be

⁴ Uniqueness is with reference to the consumption allocation (c, w) , not allocation (c, z) . There is a unique allocation (c, z) corresponding to a given allocation (c, w) only if there are no linearly dependent (redundant) securities in the market.

fully allocationally efficient, because (11) insures that the marginal rates of substitution of wealth between any two states are the same for all investors i .⁵

The *equilibrium value* of a feasible second-period payoff vector w will be denoted $V(w)$; thus if w is obtainable via portfolio z , we obtain $w = zA$ and hence

$$V(w) = V(zA) = zP = wR = zAR \tag{12}$$

In the above, R represents the not necessarily unique set of *implicit prices* of (second-period) consumption in the various states implied by P . By Farkas's Lemma, an implicit price vector is always present in the absence of arbitrage and hence in equilibrium.

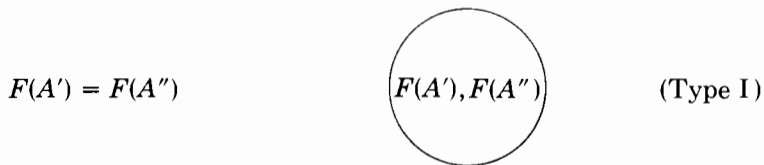
II. Feasible Allocations

In this section, we characterize the sets of possible second-period allocations w_i obtainable via different market arrangements. Recall that a market structure A is any "full" set of instruments, i.e. any set of instruments capable of allocating aggregate wealth $W = (W_1, \dots, W_n)$. Thus a market may have as few as one instrument. The set of feasible second-period consumption allocations $w = (w_1, \dots, w_j)$ obtainable via market structure A will be denoted $F(A)$, i.e.

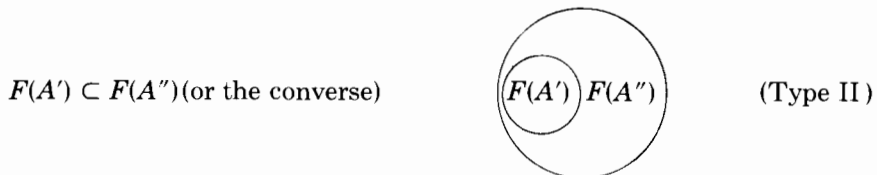
$$F(A) \equiv \{w \mid w_i \geq 0, w_i = z_i A, \sum_i z_{ij} = Z_j, \text{ all } j\}$$

Note that this definition makes no reference to, and is therefore completely independent of, endowments. It merely summarizes all conceivable allocations of second-period consumption via the securities that comprise A .

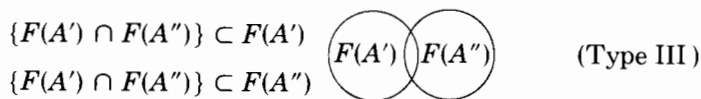
In comparing two market structures A' and A'' with respect to feasible allocations, we first observe that $F(A') \cap F(A'')$ is always nonempty since holding the "market portfolio", i.e. an equal proportion of every instrument, for example, is always feasible. Thus, there are three possibilities; either



or



or



⁵ It is sufficient (but not necessary) for (11) to obtain that the security market be complete, i.e. that the rank of A be n or that the market be equivalent to an Arrow-Debreu [1, 9] market. Clearly, (11)

A necessary, but not sufficient, condition for a market structure change to be of Type I or *feasibility preserving* is that A' and A'' have the same rank, i.e. $\text{Rank}(A') = \text{Rank}(A'')$. Similarly, a necessary, but not sufficient, condition for a change from A' to A'' to be of Type II or *feasibility expanding* is that $\text{Rank}(A') < \text{Rank}(A'')$; the reverse change will be referred to as *feasibility reducing*. However, if $\text{Rank}(A) < n$, then clearly $F(A) \subset F(A_n)$, where A_n is a complete (financial) market.

A Type III or *feasibility altering* comparison may be illustrated by a simple example:

	A'			\longleftrightarrow	A''			
	1	2	2		1	1	3	
	2	2	3		2	3	2	
W	300	400	500		W	300	400	500

For instance, a payoff $w = (15, 18, 24)$ for the three states is attainable in A' (buy 3 and 6 shares, respectively, of the two instruments) but not in A'' . Conversely, a payoff $w = (15, 18, 33)$ is feasible in A'' (buy 9 and 3 shares, respectively, of the two instruments) but not in A' .

A sure way to obtain a feasibility expanding change is to make a finer and finer breakdown of *existing* instruments into an ever larger set of linearly independent securities. Type III changes, however, remind us that sheer numbers are not the end-all: two instruments alone may be able to accomplish some of the things that a million (linearly independent) instruments cannot. The relation \subseteq among feasible allocations induces only a partial ordering of financial markets.

In the sequel, an instrument j will be termed *reproducible, redundant, or spanned in market A* if its payoff pattern a_j is obtainable via a linear combination (or portfolio) of other instruments in A .

III. Endowment Neutrality

The welfare implications of moving from market structure A' to market structure A'' , or the comparative implications of moving from A^0 to A' versus from A^0 to A'' , depend not only on the feasible allocations $F(A')$ and $F(A'')$ but also on the accompanying "endowment effects." The latter effects arise because changes in the market are not the result of divine action but must be accomplished via human intervention. Consequently, there are three reasons why genuine changes in individual endowments are *inextricably linked* to many of the more common types of market structure changes. First, virtually all market structure changes (announcements) occur when the market is closed or when trading has been suspended. Second, market structure changes often involve the substitution of new securities for old ones in such a way that the individual investor simply ends up with a different portfolio (payoff) without any direct say in the matter (except

also occurs when *all* consumer-investors are *completely* identical. A third set of well-known sufficient conditions which imply (11) is the following: at least two securities in the market, one of which is risk-free; homogeneous beliefs; additive utility, with the second period's utility function being a member of the HARA-class, the exponent being the same across all consumer-investors (except in the negative exponential case). (See footnote 11 for a specification of the HARA-class.)

perhaps the opportunity to vote for or against management's wishes).⁶ Finally, many market structure changes involve only a handful of securities, often only two, without any opportunity to even partially neutralize the impact via changes in other holdings. From the individual's perspective, then, market structure changes typically have a built-in initial effect, absorbed by his endowed portfolio, that is in every sense *imposed*. For example, in a simple merger of two companies X and Y , the separate payoff patterns of the common of X and of Y become combined into a new single pattern. But unless the investor happens to hold the two securities in their market portfolio proportions (at the time of the announcement), the accompanying exchange of shares generally also affects each individual's endowment pattern (\bar{c}_i, \bar{w}_i) . For example, suppose there are three states and that the per share payoffs for Companies X and Y in market A' are (2, 3, 4) and (1, 3, 6), respectively, and that each firm has 10 shares outstanding. If Company X now offers one of its shares, say, in exchange for one share of Company Y and a nonsynergistic merger is consummated, the new market A'' will have 20 shares of Company X , each with payoff (1.5, 3, 5), and no shares of Company Y . But the endowment of an investor with two shares of X and one share of Y in A' , a total of (5, 9, 14), has now, through the exchange of shares, been *shifted* to an endowment of (4.5, 9, 15) in market A'' . Similarly, the sale of a subsidiary by one company to another clearly alters the future payoffs of at least two securities. *Thus, our concern here is with those endowment effects which, avoidably or unavoidably, arise as a result of the replacement of existing instruments with new ones in the implementation of a market structure change.*

More precisely, a change in the market structure under pure exchange is brought about by the substitution, removal, and/or addition of securities, including the numeraire commodity (cash), in such a way as to leave the supply of aggregate consumption, i.e. $\sum_i \bar{c}_i$ and W , unchanged. In the present context, all such changes are completed before the market opens and may be decentralized, i.e. instituted by individual firms, or possibly centralized (an example of the latter would be the opening or closing of option markets). As noted, decentralized market structure changes, besides being the more common, have the property that they affect only a relatively small subset of the securities available in the market. But since each security typically represents a complex payoff pattern across the states, changes in the market structure will almost inevitably alter individual endowments across states.

It will be useful to distinguish between two kinds of endowment effects in connection with market structure changes. To identify the first type of effect, we define

Strong endowment neutrality. A change from market structure A' to market structure A'' exhibits strong endowment neutrality if the endowed consumption

⁶ A number of institutional arrangements designed to counteract endowment effects can be observed. For example, bond covenants and various "me-first" rules are designed to protect debtholders against certain actions by shareholders and/or management, such as the issuance of new debt with the same priority as existing debt (see e.g. Fama and Miller [13], Jensen and Meckling [24], and Fama [12]). While such protection is clearly possible in special cases, endowment shifts in general are rather difficult to insure against, as exemplified by some of the illustrations which follow.

patterns in the two markets are identical, i.e. if

$$(\bar{c}'_i, \bar{w}'_i) = (\bar{c}''_i, \bar{w}''_i), \quad \text{all } i \quad (13)$$

As defined, strong endowment neutrality requires not only that each consumer-investor's initial claims to current consumption remain unchanged between the two markets but that his initial claims to end-of-period wealth *in each state* be the same in the two markets. Any departure from (13) constitutes an endowment shift and will in general (as we shall see) have significant welfare and price effects.

Consider first a change from A' to A'' . In general, some instruments in J' will then be continued in the new set J'' and some will disappear. Denote the disappearing subset by $J'_c \subseteq J'$ and the subset of new instruments by $J''_c \subseteq J''$. Since endowment holdings of securities $j \notin J'_c$ continue unchanged in A'' , we observe that (13) can now be guaranteed, for an *arbitrary* endowment pattern, only if the payoff from each security $j \in J'_c$ can be replaced by a linear combination of instruments in J''_c . Thus, a necessary and sufficient condition for strong endowment neutrality to be *attainable* in conjunction with a change from A' to A'' is that $F(A'_c) \subseteq F(A''_c)$, where A'_c is the subset of disappearing payoffs and A''_c the subset of payoffs which replaces them. Of course, *the attainability* of strong endowment neutrality is no guarantee that it will be *implemented*.

One implication of the preceding is that movements from one complete market to another, via decentralized changes in the market structure, are generally *not* accompanied by strong endowment neutrality. In the same vein, we observe that $F(A') \subseteq F(A'')$ may or may not be accompanied by $F(A'_c) \subseteq F(A''_c)$. Finally, even if $F(A'_c) \subseteq F(A''_c)$ should hold in a change from A' to A'' , the reverse change from A'' to A' clearly need *not* be accompanied by $F(A''_c) \subseteq F(A'_c)$.

The following are examples of market structure changes that *generally do not* satisfy strong endowment neutrality:

1. 100% nonsynergistic mergers.
2. The refinancing of bonds (prior to maturity) with new bonds of a different maturity.
3. The calling of convertible bonds and replacing them with regular bonds of the same (or a different) maturity.⁷
4. The conversion of convertible securities into common stock.
5. The sale of a subsidiary or division by one company to another.
6. The closing of option markets, with settlements in cash and/or the underlying securities.

The following types of market structure changes, on the other hand, *do* preserve strong endowment neutrality:

1. Nonsynergistic (pro rata) corporate spinoffs when applicable bonds remain risk-free.
2. The issuance, on a pro rata basis, of bonds (risk-free or subordinated) or preferred stock in exchange for common stock placed in treasury.
3. The opening of option markets.

⁷ Note that example 3 and 4 also typically affect the payoff pattern on the common stock and other derivative securities.

Strong endowment neutrality is clearly a very demanding requirement. A less restrictive notion of neutrality which more closely captures our intuitive sense of this concept is that the exchange of endowment securities and "cash" transfers to "even" in terms of value. More precisely,

Weak endowment neutrality. A change from market structure A' to market structure A'' exhibits weak endowment neutrality if the values of the endowments, provided there is a common implicit equilibrium price structure R , are identical in the two markets, i.e. if

$$\bar{c}'_i + \bar{z}'_i P' = \bar{c}'_i + \bar{w}'_i R = \bar{c}''_i + \bar{w}''_i R = \bar{c}''_i + z''_i P'', \quad \text{all } i \quad (14)$$

where $R > 0$ satisfies $A'R = P'$ and $A''R = P''$.

A common equilibrium implicit price vector can of course not always be found for two markets, when $F(A') \neq F(A'')$, such a situation is the exception rather than the rule. But when a common R is present in equilibrium, every consumption pattern in A'' can be valued via P' and every pattern in A' via P'' so that the notion of *equivalued* endowments becomes both meaningful and useful.

IV. Comparisons of Equilibria

In comparing different market structures, the comparison which is ultimately relevant is that which compares allocations actually attained, i.e. equilibrium allocations. Furthermore, our primary concern is with the welfare implications of such (equilibrium) allocations. Using (4), we denote investor i 's equilibrium expected utility in market structure A'' by u''_i and his equilibrium expected utility in market structure A' by u'_i . A comparison of any given equilibrium in market A'' with some equilibrium in some other market A' must then yield one of four cases:

$$u''_i \geq u'_i, \text{ all } i, u''_i > u'_i, \quad \text{some } i \quad (\text{Pareto dominance}) \quad (i)$$

or

$$u''_i = u'_i, \text{ all } i \quad (\text{Pareto equivalence}) \quad (ii)$$

or

$$u''_i > u'_i, \text{ some } i, u''_i < u'_i, \quad \text{some } i \quad (\text{Pareto redistribution}) \quad (iii)$$

or

$$u''_i \leq u'_i, \text{ all } i, u''_i < u'_i, \quad \text{some } i \quad (\text{Pareto inferiority}). \quad (iv)$$

The task at hand, then, is to identify the conditions under which each of these cases, as well as combinations of these cases, will occur. All comparisons are *contemporaneous* in the sense that they compare welfare under market structure A'' to what it would be if A' were in use instead.

The subset of consumption allocations (c, w) which, for a given set of beliefs and preferences $\{\pi, U\}$ satisfying the assumptions in Section I, are obtainable as competitive solutions to system (5)-(10) for all feasible endowments (i.e. all endowments satisfying (3)) will be denoted $F^*(A)$. Under our assumptions, all allocations in this set are allocationally efficient. Note, however, that since $(\bar{c}_i, \bar{w}_i) = 0$ is not permissible, $F^*(A)$ is a proper subset of all allocations that are allocationally efficient with respect to the set $\{\pi, U\}$.) Since $F(A'') = F(A')$

implies that the set of feasible endowments are the same in the two markets, it also implies that $F^*(A') = F^*(A'')$ for any set of beliefs and preferences.

We now state a useful intermediate result.

LEMMA. Consider a feasibility expanding market structure change, i.e. let A'' and A' be financial markets such that

$$F(A') \subset F(A'') \quad (\text{Type II})$$

Then there exists beliefs, preferences, and endowments which yield either (i) Pareto dominance, (ii) Pareto equivalence, or (iii) a Pareto redistribution, but not (iv) Pareto inferiority.

An illustration which yields Pareto dominance may be found in Section IX; while based on strong endowment neutrality, it is readily altered to accommodate endowment shifts. An example for which Pareto equivalence or a Pareto redistribution obtains is that of homogeneous probability assessments and $U_{is}(c_i, w_{is}) = u_i(c_i) + \alpha_i w_{is}^\gamma$ for all i (with $\alpha_i > 0$ and $\gamma < 1$); in this case, all investors, as is well-known, always hold "the market portfolio" of all assets and every market A yields full allocational efficiency. Under weak endowment neutrality, the optimal market portfolio proportions under A' and A'' are unchanged for any given investor, yielding Pareto equivalence, while shifts in endowments give rise to a Pareto redistribution.

The following corollary is immediate:

COROLLARY 1. Consider a feasibility altering (Type III) market structure change. Then one can find beliefs, preferences, and endowments that yield either Pareto superiority, Pareto equivalence, a Pareto redistribution, or Pareto inferiority, i.e. no welfare effect is excluded.

It should be noted that "large" transfers of endowment are always redistributive, independently of any change in the market structure. The other possible outcomes must therefore be based on at most "small" shifts in endowments.

V. Equivalent Markets

When two market structures A' and A'' provide the same allocational possibilities, i.e. $F(A') = F(A'')$, they may be thought of as equivalent. But allocational equivalence does not necessarily imply that everyone's expected utility will be unchanged as the result of a shift from one market to the other. For example, we have already observed that an endowment (\bar{c}_i, \bar{w}_i) may *not* be transferable from one market to another that is allocationally equivalent. In addition, multiple equilibria may occur and must be dealt with. Finally, Pareto equivalence may occur for some subsets of preferences and beliefs but not others. Consequently, we shall need to distinguish between several types of equivalence: weak and strong, global and local.

Our first result concerns weak, global market equivalence, i.e. what we can say independently of preferences, beliefs, and endowments.

THEOREM 1. (*Weak global market structure equivalence.*) A necessary and sufficient condition for a change from A' to A'' (and the converse) to yield either

Pareto equivalence or a Pareto redistribution for arbitrary beliefs, preferences, and endowment shifts is that feasibility is preserved, i.e. that

$$F(A'') = F(A') \quad (\text{Type I})$$

Proof: The equilibrium allocations (c', w') and (c'', w'') are such that $(c', w') \in F^*(A')$ and $(c'', w'') \in F^*(A'')$. But in Type I comparisons we have, as already noted, $F^*(A') = F^*(A'')$, which in turn implies Pareto equivalence or a Pareto redistribution independently of endowments. This establishes sufficiency.

To show necessity, suppose to the contrary that $F(A'') \neq F(A')$, which in turn means that the comparison is either of Type II or of Type III. But by the Lemma and Corollary 1, there now exist beliefs and preferences for which Pareto dominance or Pareto inferiority obtain not only under endowment neutrality but for sufficiently small endowment shifts. Thus, $F(A') = F(A'')$ is indeed required for Pareto equivalence or a Pareto redistribution alone to be possible when preferences and beliefs are unrestricted.

Theorem 1 reminds us that while Pareto dominance cannot occur in market comparisons of Type I, redistributive effects are not ruled out. The next goal is to identify conditions which insure that each equilibrium allocation (c', w') in A' is also an equilibrium allocation in A'' and conversely. This is a much stronger implication than Theorem 1 offers and requires the presence of strong endowment neutrality as well.

THEOREM 2. (*Global equilibrium correspondence.*) *A necessary and sufficient condition for an equilibrium in A' to be an equilibrium in A'' , and conversely, for arbitrary beliefs and preferences is that feasibility in the two markets is identical and strong endowment neutrality holds, i.e. that*

$$F(A'') = F(A') \quad (\text{Type I})$$

and

$$(\bar{c}'', \bar{w}'') = (\bar{c}', \bar{w}') \quad (13)$$

Proof: Let (c', w') be an equilibrium allocation under A' . $F(A'') = F(A')$ then implies that there exists a security portfolio z'' in A'' such that

$$(c'_i, z''_i A'') = (c'_i, w'_i), \quad \text{all } i \quad (15)$$

and that there exists weights e_{jk} such that

$$a''_{js} = \sum_{k \in J'} e_{jk} a'_{ks} \quad \text{all } s, \quad \text{all } j \in J''$$

But this in turn implies that the allocation (c'', z'') satisfies equilibrium conditions (5)-(7) and (9)-(10), with

$$\lambda'' = \lambda', \quad \delta'' = \delta' \quad (16)$$

$$P''_j = \sum_{k \in J'} e_{jk} P'_k \quad \text{all } j \in J'' \quad (17)$$

and

$$P''_{is} = P'_{is}, \quad \text{all } i \text{ and } s \quad (18)$$

In view of (15), (18) also means that total expenditures are the same in the two markets, i.e.

$$c_i'' + z_i''P'' = c_i' + z_i'P', \quad \text{all } i \tag{19}$$

while (13) guarantees that endowments as well are unchanged in value. That is,

$$\bar{c}_i'' + \bar{z}_i''P'' = \bar{c}_i' + \bar{w}_i''P_i'' = \bar{c}_i' + \bar{w}_i'P_i' = \bar{c}_i' + \bar{z}_i'P, \quad \text{all } i \tag{20}$$

for each equilibrium that may occur in A' . By symmetry, each equilibrium in A'' is also an equilibrium in A' .

The necessity of $F(A'') = F(A')$ derives from the fact that, without it, (c', w') need not be feasible in A'' and (c'', w'') need not be feasible in A' . Suppose now that (13) does not hold and consider the feasible allocation (c'', z'') satisfying (15); as noted, this allocation also satisfies (5)–(7), (9)–(10), (18), and (19). But in view of (18), the inner equality in (20) need not hold in the absence of (13); even if it should hold for *some* equilibrium pair of shadow price vectors P_i'' and P_i' , it would not hold for other pairs in the case of multiple equilibria (which are not ruled out since beliefs and preferences are unrestricted). This concludes the proof.

In view of Theorem 2, the only way to guarantee a unique equilibrium in A' is to assume that

$$(c', w') = (\bar{c}', \bar{w}') \tag{21}$$

i.e. that endowments in market A' constitute an equilibrium allocation in that market. Furthermore, the only way that same allocation can now be guaranteed in market A'' in Type I comparisons, independently of preferences and beliefs, is for it to be thought about via the new endowment (\bar{c}'', \bar{w}'') since trading cannot be counted on to regenerate (c', w') when multiple equilibria are possible. In Type II reverse and in Type III comparisons, (\bar{c}', \bar{w}') need not be feasible in A'' ; in Type II changes, (\bar{c}', \bar{w}') need not be optimal in A'' . Consequently, $F(A') = F(A'')$ is also necessary to insure that $(c'', w'') = (\bar{c}', \bar{w}')$. This gives

THEOREM 3. (*Strong global market structure equivalence*). *A necessary and sufficient condition for a change from A' to A'' to yield Pareto equivalence for arbitrary beliefs and preferences is that*

$$F(A'') = F(A') \tag{Type I}$$

$$(c', w') = (\bar{c}', \bar{w}') \tag{21}$$

and

$$(\bar{c}'', \bar{w}'') = (\bar{c}', \bar{w}') \tag{13}$$

i.e. that payoff feasibility is identical, the endowments in A' are efficient, and there is strong endowment neutrality.

Perhaps the most noteworthy thing about Theorem 3 is that, given both $F(A') = F(A'')$ and (21), the presence of *weak* endowment neutrality may still bring about a *redistribution of welfare* when there are multiple equilibria.⁸

⁸ Whether the economy can be “nudged” toward a particular equilibrium (in the presence of alternate equilibria) is an intriguing question about which the current theory of adjustment processes has little to offer.

In the proof of Theorem 2, we found that $F(A'') = F(A')$ implies (15)–(19) but not necessarily (20). Suppose, however, that we have weak endowment neutrality, i.e. that (14) holds, and that the equilibria in A' and A'' are unique for the beliefs and preferences at hand. Then (20) is certain to hold so that (c', w') is the only equilibrium in A'' .

If equilibrium in A' were not unique, (14) would not hold for \bar{w}_i'' for a different price vector P' unless (13) were also valid. Similarly, unless the constructed equilibrium in A'' based on the chosen (weakly neutral) endowment is unique, there would not necessarily be an equilibrium in A' for each equilibrium in A'' . Thus we obtain

THEOREM 4. (*Strong local market structure equivalence.*) *A necessary and sufficient condition for markets A' and A'' to yield Pareto equivalence for beliefs and preferences consistent with the requirement that equilibrium in each market be unique is that*

$$F(A'') = F(A') \quad (\text{Type I})$$

and that there is a common implicit price vector R for both P' and P'' such that

$$\bar{c}_i'' + \bar{w}_i'' R = \bar{c}_i' + \bar{w}_i' R, \quad \text{all } i \quad (14)$$

i.e. that payoff feasibility is identical and weak endowment neutrality holds.

In a change from A' to A'' , weak endowment neutrality can be thought of as based on the equilibrium prices that prevail in the first market A' . These would not necessarily be known prior to trading when the substitution of securities must take place. Weak endowment neutrality is thus somewhat difficult to visualize in practice. One possible scenario would be to assume that endowments in A' already represent an equilibrium allocation achieved in “earlier” trading that revealed prices P' . (14) and (17) then provide the mechanism to implement weak endowment neutrality so as to achieve an equivalent equilibrium in A'' at least for Type I changes. This leads to the following (weaker) statement of existence.

COROLLARY 2. Assume that (\bar{c}', \bar{w}') represents an equilibrium allocation in A' supported by the price vector P' . A necessary and sufficient condition for an equivalent equilibrium allocation to *exist* in A'' for all beliefs and preferences is that

$$F(A'') = F(A') \quad (\text{Type I})$$

and

$$\bar{c}_i'' + \bar{w}_i'' R = \bar{c}_i' + \bar{w}_i' R, \quad \text{all } i \quad (14)$$

where $R > 0$ and $A'R = P'$.

As already noted, (14), as opposed to (13), does not guarantee that the equivalent equilibrium will be attained when multiple equilibria exist. However, Corollary 2, by in effect sidetracking the endowment effect, seems to come closest to capturing previous results on capital structure indifference (e.g. Modigliani and Miller [33], Stiglitz [49], and Fama [12]), all of which have paid minimal attention to endowment effects. But to assume that no trading is required in A' , which is what is needed for P' to be known and (14) to be implemented, is a strong

assumption indeed. Relaxing these requirements, however, means, as we have seen, that equivalent allocations are far from guaranteed (how do we get P' ?) and this, in turn, implies that *redistributive effects generally do accompany even those market structure changes that leave the allocational possibilities unaltered*. The same is true if we return to Theorem 3, recalling that strong endowment neutrality, as a practical matter, is generally unattainable (recall Section III). Decentralized substitutions of (subsets of) securities will thus generally have redistributive effects, in complete as well as incomplete markets, even when they are feasibility preserving.⁹

VI. Market Structure Dominance

The central feature of feasibility preserving changes in the market structure is that welfare effects are at most redistributive—and that redistribution is *unavoidable* more commonly perhaps than previously thought. When we turn to Type II or feasibility expanding changes, i.e. the case when $F(A') \subset F(A'')$, Pareto dominance is of course possible, as we saw in the Lemma. Our first result confirms that everyone cannot be worse off in A'' .

THEOREM 5. (*Weak global market structure dominance.*) *A necessary and sufficient condition for a change from A' to A'' to yield either Pareto dominance, Pareto equivalence, or a Pareto redistribution for arbitrary beliefs, preferences, and endowment shifts is that the change be feasibility expanding, i.e. that*

$$F(A') \subset F(A'') \tag{Type II}$$

To show sufficiency, we first observe that $(c'', w'') \in F^*(A'')$ and that $(c', w') \in F(A'')$ since $F(A') \subset F(A'')$. Thus, Pareto inferiority is ruled out since (c'', w'') cannot be Pareto dominated by an allocation in $F(A'')$. In addition, by the Lemma there are indeed beliefs, preferences, and endowment shifts under which each of situations (i)–(iii) obtains.

The necessity of $F(A') \subset F(A'')$ follows from the observation that if this condition does not hold, we must either have a Type I comparison, which cannot generate Pareto dominance by Theorem 1, or a Type III comparison, which by Corollary 1 does not rule out Pareto inferiority when endowment shifts are sufficiently small.

Similarly, Theorem 3 translates directly to

THEOREM 6. (*Strong global market structure dominance.*) *A necessary and sufficient condition for a change from A' to A'' to yield either Pareto dominance or Pareto equivalence for arbitrary preferences and beliefs is that*

$$\begin{aligned} F(A') \subset F(A''), & \tag{Type II} \\ (c', w') = (\bar{c}', \bar{w}') & \tag{21} \end{aligned}$$

and

$$(\bar{c}'', \bar{w}'') = (\bar{c}', \bar{w}') \tag{13}$$

⁹ In the case of mergers, the redistributive effect will generally be exacerbated by the common practice of paying a “premium” for the target’s shares.

i.e. that feasibility is expanded, the endowments in A' are efficient, and there is strong endowment neutrality.

A comparison of Theorems 5 and 6 reveals that *the only way that redistributive effects can be avoided with certainty in moving to an allocationally richer market is to guarantee that the equilibrium allocation in A' is the initial position in A''* . This requires both (21) and (13); if either is absent, $(\bar{c}'', \bar{w}'') \neq (c', w')$ and there will then be some preferences and beliefs for which not only $(c'', w'') \neq (c', w')$ but which leave some consumer-investors worse off in A'' than they would be in A' .

As in Theorem 4, strong endowment neutrality can be relaxed to weak endowment neutrality in Theorem 6 but only under very special conditions. First, to expand the feasible portfolios for any initial wealth level, the implicit price structure in the two markets must be the same. Second, equilibrium in each market must be unique (since weak endowment neutrality generally does not hold, for given endowments, for more than one price system). Formally,

THEOREM 7. (*Strong local market structure dominance.*) *A necessary and sufficient condition for a change from A' to A'' to yield either Pareto dominance or Pareto equivalence for beliefs and preferences consistent with the requirement that equilibrium in each market be unique is that*

$$F(A') \subset F(A'') \quad (\text{Type II})$$

and that there exists a common implicit price vector R for both P' and P'' such that

$$\bar{c}_i'' + \bar{w}_i'' R = \bar{c}_i' + \bar{w}_i' R, \quad \text{all } i \quad (14)$$

i.e. that feasibility is expanded and there is weak endowment neutrality.

Finally, the following result considers movements in the opposite direction, from less constrained to more constrained markets.

COROLLARY 3. (*Local welfare preservation*) In changing from market structure A'' to market structure A' , where $F(A') \subset F(A'')$, a sufficient condition for Pareto equivalence is that

$$(c'', w_i'') = (\bar{c}'', \bar{w}'') \quad (22)$$

and

$$(\bar{c}', \bar{w}') = (\bar{c}'', \bar{w}'') \quad (23)$$

Proof: Assumptions (22) and (23) guarantee that $(c', w') \in F^*(A'')$. Since $F(A') \subset F(A'')$, every feasible $(c', w') \in F^*(A'')$ is such that $(c', w') = F^*(A')$ and hence no trading will occur.

It should be noted that when $F(A') \subset F(A'')$, strong endowment neutrality will rarely be satisfied because the greater richness of market structure A'' will be reflected in the equilibrium allocations (c_i'', w_i'') . In fact, only for those beliefs and preferences which don't take advantage of the additional opportunities provided by A'' , by holding the market portfolio, say, and certain singular cases will we find that strong endowment neutrality is not totally ruled out. *The implication of this is that market structure changes which limit feasible allo-*

cations will almost always either have redistributive effects or leave everyone worse off.

VII. Type III Changes

Market structure changes of Type I can, as we have seen, never lead to Pareto dominance. Similarly market structure changes of Type II preclude Pareto dominance in *one* (but not both) directions. In view of Corollary 1, it is then both necessary and sufficient, for sufficiently small endowment effects, to be in the domain of Type III changes in order for *any one* of the outcomes (i) through (iv) to be feasible. That is,

THEOREM 8. (*Unconstrained outcomes.*) *A necessary and sufficient condition for a change from A' to A'' to yield any one of the outcomes (i)–(iv) for arbitrary beliefs, preferences, and endowment shifts is that the change be feasibility altering, i.e.*

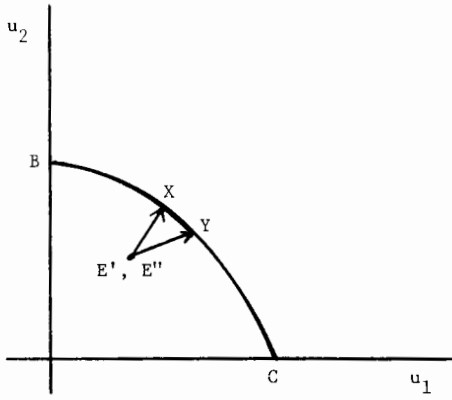
$$\begin{aligned} \{F(A') \cap F(A'')\} \subset F(A') \\ \{F(A') \cap F(A'')\} \subset F(A'') \end{aligned} \qquad \text{(Type III)}$$

When preference and beliefs are unrestricted, Theorems 1, 5, and 8 reveal that the welfare implications for each type of market structure change are not only sharply delineated but substantively different. The basic mapping from Types I–III to welfare space is both simple and informative. The absence of “strong” results for Type III comparisons, of course, does not imply that Type III changes should be viewed as rare—quite the contrary.

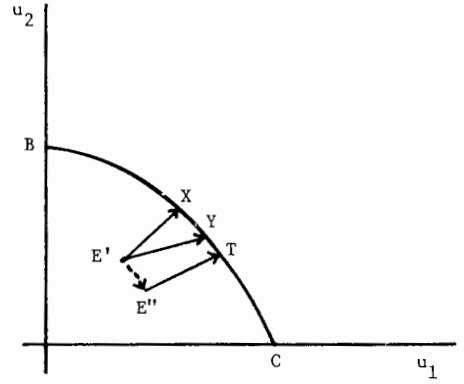
VIII. Graphic Illustrations

The preceding theorems are quite rich in content. It may therefore be useful to provide a graphic illustration of a portion of what they communicate.

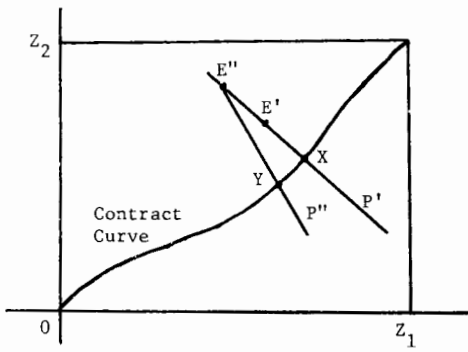
In Figures 1–2 and 4–6, curve BC depicts the set of efficient allocations, measured in expected utility u_i , in market A' when there are two individuals (numbered 1 and 2). Given a change to A'' that is feasibility preserving, as in Figures 1–2, the efficient allocations remain on curve BC (Theorem 1; this would also be true for Type II or III changes under certain preferences, beliefs, and endowments as shown in Theorems 5 and 8). If E' represents the endowment point in A' , strong endowment neutrality implies that E'' coincides with E' . Figure 1 thus illustrates the sufficiency part of Theorem 2: if X and Y are the only equilibria in A' given E' , then X and Y are also the only equilibria in A'' given strong endowment neutrality. In the absence of *strong* endowment neutrality, E'' will (generally) differ from E' (see Figure 2), causing the equilibrium in A'' to occur at T rather than at X or Y (which illustrates the necessity part of Theorem 2). Theorem 3 is now readily understood as well: In order for the equilibrium in A'' to be welfare-preserving (occur at X , say, in Figures 1 and 2), it is clear that we must have: (1) a Type 1 change; (2) E'' must coincide with E' ; and (3) E' must coincide with X .



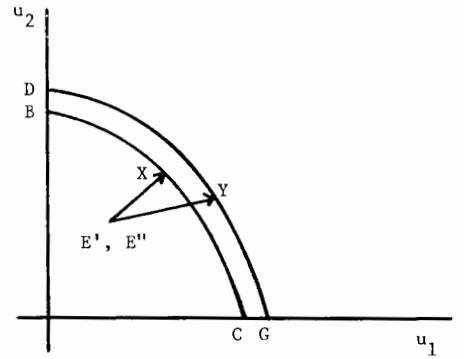
1 Type I (or II or III) change



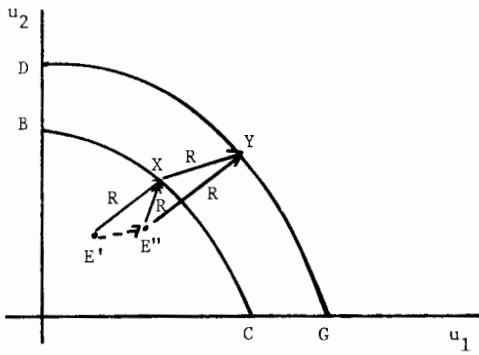
2 Type I (or II or III) change



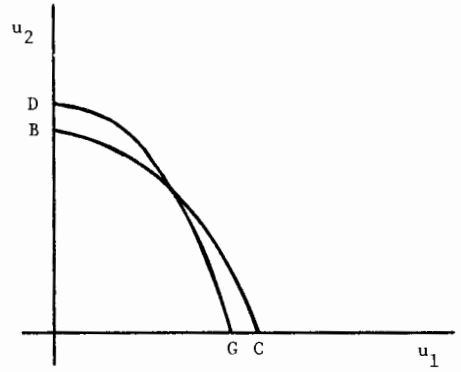
3 Type I change



4 Type II change



5 Type II change



6 Type III change

Figure 1. Type I (or II or III) change

Figure 2. Type I (or II or III) change

Figure 3. Type I change

Figure 4. Type II change

Figure 5. Type II change

Figure 6. Type III change

Figure 3 is an Edgeworth box for the two-individual, two-asset case, where Z_1 and Z_2 are the number of outstanding shares of securities 1 and 2. In a Type I change, the contract curve is unchanged. Let E' be the endowment point and X the equilibrium in A' . Under weak endowment neutrality E'' lies on the opportunity line P' so that X is then also an equilibrium in A'' . But for welfare to be preserved, there cannot be another equilibrium Y (in A'' , say), i.e. the equilibrium must be unique in both markets (Theorem 4).

In a feasibility expanding change, the set of efficient allocations either remains at BC or moves outward to curve DG (see Figures 4 and 5). In the latter case, we may end up with a Pareto redistribution (Theorem 5) even under strong endowment neutrality, as shown in Figure 4. The only way to insure a Pareto improvement in general is for E' and E'' to coincide with the equilibrium point X in A' (Theorem 6). However, as Figure 5 illustrates, a Pareto-improvement also occurs if (1) there is weak endowment neutrality and (2) equilibrium is unique in both A' and A'' (Theorem 7). In particular, the preceding is consistent with the following: in A' , X is the only equilibrium for both E' and E'' , and in A'' , Y is the only equilibrium starting from either E'' or X , with R serving as the equilibrium implicit price vector in *all* cases. Finally, in a feasibility altering change the set of efficient allocations either remains unaltered or moves from BC to the crossing curve DG , which leaves open all possibilities (Theorem 8).

IX. Welfare and Prices

Apparently beginning with Hirshleifer [23, p. 275] a number of authors have observed that value maximization by firms need not be an optimal rule in incomplete markets. While the impact on prices of market structure changes has not been addressed directly, there appear to be at least some suggestions to the effect that the equilibrium prices and welfare improvements attained by market enrichment are positively related.¹⁰ It may therefore be worthwhile to examine briefly the relation between welfare and prices in the current framework. We know from Sections V-VII, as well as from the theory of international trade (see e.g. Samuelson [43]), that shifts in endowments alter the terms of trade in an arbitrary fashion. This section will demonstrate that a change in the financial market, quite apart from any other change, also has an arbitrary effect on equilibrium prices.

Suppose that, for simplicity, there are two consumer-investors and three states. A' has two securities and A'' three, as follows:

	A'				A''		
			Z				Z
	1	1	100	1	1	1	100
	1	2	300	1	2	3	100
W	200	300	400	0	0	1	0
	W	200	300	W	200	300	400

The third security in A'' may be recognized as a call option on security 2 with an exercise price of 2. Since A'' has full rank, $F(A') \subset F(A'')$ so that a change to A'' is feasibility expanding.

¹⁰ See Sosin [48, pp. 1228, 1230, 1233] for an example.

Suppose beliefs are given by $\pi_1 = (.3, .4, .3)$ and $\pi_2 = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$ and that preferences are additive, with second-period utilities U_i and marginal utilities U'_i for selected consumption levels given by:

Individual 1:

Cons. level	126	130	150	159	165	170
U_1	0	5.7	28.5	36	40.05	42.75
U'_{1a}	1.5	1.4	.9	.75	.6	.5
U'_{1b}	1.5	1.4	.9	.7425	.582	.5
U'_{1c}	1.5	1.4	.9	.7575	.618	.5

Individual 2:

Cons. level	70	74	141	150	230	235
U_2	0	5.7	81	88.53	142.5	145.41
U'_{2a}	1.44	1.35	.9	.72	.63	.54
U'_{2b}	1.44	1.35	.891	.72	.63	.5238
U'_{2c}	1.44	1.35	.909	.72	.63	.5562

where U'_{1a} , U'_{1b} , and U'_{1c} represent three different examples (cases) of marginal utilities consistent with the given U_i and risk aversion.

If endowments are $\bar{z}'_1 = (110, 20)$ and $\bar{z}'_2 = (-10, 80)$, there would be no trading in A' since the endowment allocation is in fact an equilibrium supported by the price vector $P' = (.93, 1.59)$. This yields the second-period consumption allocations $w'_1 = (130, 150, 170)$ and $w'_2 = (70, 150, 230)$; the resulting expected utilities are $u'_1 = k_1 + 25.935$ and $u'_2 = k_2 + 77.01$ (where k_i represents the utility of first-period consumption). Note that the preceding is true for *each* of the cases a , b , and c .

If we now move to market structure A'' (leaving endowments unchanged, which is feasible and of course insures strong endowment neutrality), individual 1 will alter his endowed portfolio $\bar{z}''_1 = (110, 20, 0)$ by trading to $z''_1 = (93, 33, -27)$, which in turn yields consumption levels $w''_1 = (126, 159, 165)$ in the three states. Analogously, individual 2 will switch his portfolio from $\bar{z}''_2 = (-10, 80, 0)$ to $z''_2 = (7, 67, 27)$, which gives $w''_2 = (74, 141, 235)$. First-period consumption levels remain unchanged. Expected utilities are $u''_1 = k_1 + 26.415 > u'_1$ and $u''_2 = k_2 + 77.37 > u'_2$ so that both individuals are better off in A'' .

Note that the preceding is true in *each* of the three cases a , b , and c . The differences in marginal utilities which characterize the three cases do become reflected in the equilibrium prices, however. In Case a , the equilibrium price vector is $P''_a = (.93, 1.59, .18)$ which means that securities 1 and 2 have the *same* equilibrium prices in the two markets—even though everyone is better off in A'' . Aggregate wealth is also valued the same, i.e. $V''(W) = V'(W)$.

In Case b , however, the equilibrium price vector $P''_b = (.9216, 1.5678, .1746)$ so that securities 1 and 2 as well as aggregate wealth have *lower* equilibrium values in A'' than in A' (recall that a unit of first-period consumption serves as numeraire in both markets). Thus, in moving from A' to A'' , welfare and equilibrium prices move in *opposite* directions in this case.

In Case c , the price vector which supports the indicated equilibrium is $P''_c = (.9384, 1.6122, .1854)$ which implies $V''(a_1) > V'(a_1)$, $V''(a_2) > V'(a_2)$ and $V''(W) > V'(W)$, i.e. that equilibrium prices and welfare move in the same direction when A' is replaced with A'' .

While A'' is a complete market in the preceding example, the result does not depend on that circumstance and is readily generalizable. To summarize, we state

Remark. (Independence of prices and welfare.) In comparing equilibrium under A'' with equilibrium under A' , equilibrium prices are independent of welfare in the sense that we may have

$$u_i'' > u_i', \quad \text{all } i$$

accompanied by either $V''(W) > V'(W)$ or $V''(W) = V'(W)$ or $V''(W) < V'(W)$.

The moral of this tale of course is that the relationship between prices and welfare is a tenuous one at best. Wealth and prices are not reliable measures of welfare.

X. Value Conservation

Conservation of values across different types of financial markets has been observed to hold for two types of situations: comparisons involving strong homogeneity and highly specialized preferences and/or beliefs, and for market structure changes which are feasibility preserving, i.e. changes of Type I. The former category is exemplified by equilibrium models in which all investors end up combining a position in the "market portfolio" with borrowing or lending, as in the mean-variance capital asset pricing model (e.g. Hamada [18], Merton [29]) and in models based on subsets of time-additive, state-independent, HARA-class preferences¹¹ and homogeneous beliefs (Rubinstein [41]). In these models, investors are indifferent between *all* financial markets that contain a risk-free portfolio provided there is weak endowment neutrality since all equilibria (in terms of (c, w)) are identical. In addition, $V'(w) = V''(w)$ for all (jointly) feasible payoffs w , i.e. values are conserved; in the HARA-class case this is true even in the *absence* of weak endowment neutrality as long as all investors have the same time-discounting of utilities.

The second group of writings providing value conservation results have focussed on Type I cases, i.e. on changes which imply that $F(A') = F(A'')$. Included in this group are the works of Williams [52], Modigliani and Miller [33], Hirshleifer [22], Robichek and Myers [39], Stiglitz [49, 50, 51], Schall [44], Mossin [34], Kim, McConnell, and Greenwood [25], Fama [12], Nielsen [38], and Hellwig [21]. Perhaps the most striking aspect about these analyses is their inadequate discussion of endowment effects, especially since, as noted in Section III, such effects are often *unavoidable* (this is not to say that endowment effects are always a problem, however). Previous results, therefore, provide a woefully incomplete picture.

¹¹ The HARA-class (hyperbolic absolute risk aversion) consists of the following utility functions of wealth with the properties $u'(w) > 0$, $u''(w) < 0$ (the first over at least a finite range of positive wealth):

$$\begin{aligned} & \frac{1}{\gamma} (w + a)^\gamma \quad \gamma < 1 \text{ (decreasing absolute risk aversion)} \\ u(w) = & - \exp(\gamma w) \quad \gamma < 0 \text{ (constant absolute risk aversion)} \\ & - (a - w)^\gamma \quad \gamma > 1, \text{ a large (increasing absolute risk aversion).} \end{aligned}$$

By reference to Sections V, VI, and VII, we observe that value conservation cannot occur independently of preferences and beliefs unless there is welfare conservation, i.e. Pareto equivalence holds. By Theorem 3, this requires not only $(c', w') = (\bar{c}', \bar{w}')$ but strong endowment neutrality, which we know may be *impossible* to attain in any given market structure change. In effect, then, Theorem 3 may be restated as

THEOREM 3'. (Global value conservation.) A necessary and sufficient condition for a change from A' to A'' to yield value conservation, i.e.

$$V''(w) = V'(w), \quad \text{all (jointly) feasible } w$$

for arbitrary preferences and beliefs is that

$$F(A'') = F(A'), \quad (\text{Type I})$$

$$(c', w') = (\bar{c}, \bar{w}') \quad (21)$$

and

$$(\bar{c}'', \bar{w}'') = (\bar{c}', \bar{w}') \quad (13)$$

i.e. that the change is feasibility preserving, the endowments in A' are efficient, and there is strong endowment neutrality.

We are forced to conclude, then, that (global) value conservation is a rare phenomenon. If we limit ourselves to unique equilibrium situations, however, we can dispense with the need to start in equilibrium and relax strong endowment neutrality to weak neutrality *provided* the latter ensures that the same valuation function is operative in both markets—which is only true for *all* beliefs and preferences consistent with unique equilibria in Type I changes. In other words, Theorem 4 can also be restated as

THEOREM 4'. (Local value conservation.) A necessary and sufficient condition for a change from A' to A'' to yield value conservation, i.e.

$$V''(w) = V'(w), \quad \text{all (jointly) feasible } w$$

for arbitrary beliefs and preferences consistent with the requirement that equilibrium in each market be unique is that

$$F(A'') = F(A') \quad (\text{Type I})$$

and

$$\bar{c}'_i + \bar{w}'_i R = \bar{c}''_i + \bar{w}''_i R, \quad \text{all } i \quad (14)$$

where R is compatible with both P' and P'' , i.e. that the change is feasibility preserving and weak endowment neutrality holds.

XI. Applications

At this point, it may be useful to compare the conventional wisdom with what the preceding results have to offer concerning some of the more commonly encountered situations involving changes in the structure of the financial market. For this purpose, attention will be limited to a handful of illustrations.

Spinoffs

In a corporate spinoff, a segment of an existing corporation is set up as an independent company, and existing shareholders are simply given new shares, representing their ownership in the new entity, on a pro-rata basis (such as 1 new share and .05 bonds for 2 previous shares). Assuming that all new securities accrue to the common stockholders and that the payoffs on all other securities are unaffected, strong endowment neutrality is automatically satisfied in the absence of synergy and no portfolio choices disappear so that the market structure change is either of Type I or Type II. More precisely,

- A. If both the old and all the new securities¹² are reproducible, or the old common was unique and exactly *one* of the new securities is unique, the market structure change is feasibility preserving. In this case, there is "no impact," that is, either
1. both values and welfare are unaffected if endowments were in fact an efficient allocation before the new securities arrived (Theorem 3), or
 2. if the endowments were not an efficient allocation when the new securities arrived, the set of possible equilibria after trading is the same as it was before (Theorem 2).
- B. If the old common was reproducible or unique and two or more of the new securities are unique, the market structure change is of Type II. In this case, we obtain
1. that the possibility of redistributive effects occurs only if endowments were not efficient allocations before the new securities arrived (Theorems 5, 6); otherwise we find
 2. that investor welfare is either unchanged or improved.

With the exception of possibility B.1., we can thus conclude that (simple) nonsynergistic spinoffs are at worst neutral and generally beneficial in terms of investor welfare in the context of the present model. One could certainly argue that, *ceteris paribus*, they should be welcomed rather than discouraged.

Mergers

A merger may be thought of as the opposite of a spinoff. Since we are dealing with pure exchange, it will be necessary to restrict our attention to nonsynergistic mergers which involve an exchange of securities; any cash payments must be among investors only (the exchange need not involve 100% of the outstanding quantity of any given security, however). In terms of feasible allocations, a merger involving at most common stock and risk-free debt is either feasibility preserving or feasibility reducing. If other types of securities are involved (as in Stiglitz [50], Azzi [2], and Scott [46]), Type III changes are clearly possible while a Type II change will result only if the new entity issues a richer set of securities than that which had been floated by the companies in question as separate entities. Most previous studies, however (e.g. Mossin [34], Myers [37], Mueller[36], Levy and

¹² Note that if Company X spins off a division Y by issuing common shares, the "new" shares include not only Company Y common but Company X common and possibly other securities issued by Company X—in fact all securities whose gross payoff vectors $Z_i a_i$ are affected are in fact "new" securities after the spinoff and "old" securities prior to the spinoff.

Sarnat [26], and Nielsen [38]), have focused on 100% mergers between companies having only common stock and risk-free debt outstanding, concluding, in the main, that the consequences of mergers are "neutral." For this reason, we shall limit the present discussion to that case as well. However, even in this case, the number of factors affecting the welfare consequences is perhaps surprising. Strong endowment neutrality, for example, would *rarely* be satisfied, generally occurring only when investor endowments happen to contain the merged securities in their market portfolio proportions prior to the merger announcement.

C. The market structure change is of Type I only if (1) the old securities are reproducible (in which case the new security is also reproducible) or (2) if exactly one of the previous securities is unique (in which case the new security is also unique). To conserve values and welfare we must also have either

1. endowments that represent a premerger equilibrium and strong endowment neutrality (Theorems 3 and 3'), or
2. weak endowment neutrality and a unique equilibrium in both the pre- and postmerger market (Theorems 4 and 4').

In all other cases, the merger will have redistributive effects and value conservation will typically be absent.

D. The market structure change will be feasibility reducing if both (or more) of the old securities are unique (in which case the new security will either be reproducible or unique). Value and welfare conservation can now be assured only if

1. premerger endowments represent a premerger equilibrium and strong endowment neutrality is achieved by some miraculous coincidence (Corollary 3).

In all other cases, the merger will, except in special cases, have redistributive effects or leave everyone worse off and be accompanied by an absence of value conservation.

The preceding presents a considerably gloomier picture of the welfare effects of nonsynergistic mergers than previous studies, in which "neutrality" and value conservation have been the central features. Most of these studies have made assumptions with respect to preferences or beliefs which are sufficiently strong to yield two-fund separation, a property which leaves investors indifferent between all market structures as long as they contain a risk-free portfolio and which also generally conserves values (e.g. Mossin [34], Levy and Sarnat [26]). Other studies have been limited to Type I mergers (e.g. Nielsen [38]). It is noteworthy that the necessity of either Condition C.1. or Condition C.2. for this case has been either overlooked or implicitly assumed.

Since simple mergers have zero welfare effect under investor heterogeneity only in the presence of C.1., C.2., or D.1., the neutrality of nonsynergistic mergers would appear to be the exception rather than the rule. Pareto improvements being impossible, the prevalent result will be a welfare redistribution or a welfare reduction. In view of the tendency to overcompensate the "target" company's shares in terms of premerger prices, redistribution may be the typical outcome, with those who have proportionately large premerger holdings in the target

company's common tending to benefit, primarily at the expense of those who have large proportionate holdings in the "acquiring" company's shares.¹³

Option Markets

The possible welfare implications from opening markets in (European) options are identical to those resulting from (simple) corporate spinoffs.¹⁴ That is, strong endowment neutrality is trivially satisfied, and the resulting market structure change will be either feasibility preserving or feasibility expanding. Thus the introduction of option trading leads either to Pareto equivalence, to a Pareto redistribution, or to a Pareto improvement, with redistribution possible only if endowments are not an efficient allocation when the options are introduced.¹⁵

Capital Structure Changes

One of the most enduring propositions of modern finance, in the kind of perfect market environment we have assumed, is that the value of the firm is independent of its capital structure (Modigliani-Miller [33], Stiglitz [49]). In application, this statement has been limited to mixtures of debt and common stock (with the debt assumed to be risk-free whenever the financial market is incomplete or investors exhibit reasonable heterogeneity). Even so, as Theorems 3' and 4' remind us, the validity of this classical statement depends critically on (as far as I know) previously unidentified preconditions: either the market must be in equilibrium before the exchange of (equivalent) securities (in such a way that state-by-state payoffs are preserved) or, if not, the equilibrium must be unique and weak endowment neutrality must be attained.

The findings of the present paper, of course, enable one to make comparisons between capital structures of unlimited variety, i.e. involving not only common and regular debt, but also preferred stock, risky debt, subordinated debt, convertibles, and warrants, including various and sundry protective covenants. A full analysis of these possibilities is well beyond the scope of this paper. However, consider for a moment the choice of the level of risky debt to be combined with equity in an incomplete market setting, which has been the subject of a large number of studies (e.g. Smith[47], Baron [3, 4], Milne [30, 31], and Hagen [14]). In this case, moving to the firm's alternative financing method may lead to a market structure change of Type I (if none of the (four) securities is unique), or Type II (e.g. if only the securities in the alternative are unique), or Type II reversed (e.g. if only the securities in the existing capital structure are unique instead) or Type III (if at least one security in each alternative is unique). On top of this, endowment effects must, as we have seen, be considered. Clearly, pre-

¹³ For empirical studies providing varying degrees of support for this statement, see Mandelker [28], Dodd [10], and Dodd and Ruback [11].

¹⁴ For previous studies on this subject, see Schrems [45], Ross [40], and Hakansson [16, 17].

¹⁵ It is noteworthy that the assumptions behind the option pricing models based either on arbitrage arguments (e.g. Black and Scholes [5], Cox and Ross [8]) or on special preference functions (e.g. Rubinstein [42], Brennan [7]) also imply that the options in question are redundant, i.e. their introduction give rise to a Type I change, that is, no change in the market structure or a change without impact on welfare.

dicting the welfare and wealth effects of various capital structure alternatives is not always a simple task—even though the relevant determinants are readily identified.

XII. Concluding Comments

Since the central findings were summarized at the beginning of the paper, they will not be repeated here. However, it is worth restating that the price and welfare effects caused by changes in the financial market are attributable to three principal determinants:

- (1) whether allocational feasibility has been preserved, expanded, reduced, or shifted;
- (2) whether there is strong endowment neutrality, weak endowment neutrality coupled with unique equilibria, or neither; and
- (3) whether or not the endowments in the “old” market would have represented an equilibrium had trading in that market been resumed.

The preceding implies, for example, that the requirements which give rise to value conservation in comparisons between different market structures are considerably more demanding than previously recognized. Since Pareto redistributions are difficult to avoid in the absence of either endowment neutrality or endowments that would have been efficient upon the resumption of trading in the “old” market, decentralized changes in the structure of the capital market are particularly likely to give rise to redistributive effects, even in movements between complete markets. There are two rather clearcut reasons for this: first, endowment neutrality may simply not be feasible, and even if feasible, there are no obvious forces seeking its attainment. Second, there is little validity in the hypothesis that the previous day’s (week’s) final allocation still represents an equilibrium when the market reopens for trading. Thus, the opening of option markets, while not immune to redistributive effects, represents perhaps the simplest and easiest means to achieve Pareto improvements in the welfare of consumer-investors. Nonsynergistic corporate spinoffs also tend to have positive effects while nonsynergistic mergers on balance have negative welfare effects.

The next step is to bring taxes into the picture, a task which is currently underway.

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