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Gains From Diversifying Into Real Estate: Three Decades of Portfolio Returns Based on the Dynamic Investment Model

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This paper compares the investment policies and returns for portfolios of stocks and bonds with and without up to three categories of real estate. Both domestic and global settings are examined, with and without the possibility of leverage. The portfolios were generated via the dynamic investment model based on the empirical probability assessment approach applied to past (joint) realizations of returns, both with and without correction for "smoothing" in the real estate data series. Our principal findings are: (1) the gains from adding real estate, on a semi-passive (equal-weighted) basis, to portfolios of either U.S. or global financial assets were relatively modest; in contrast, (2) the gains from adding real estate to the universe of U.S. financial assets under an active strategy were rather large (in some cases highly statistically significant), especially for the very risk-averse strategies; (3) the gains from adding U.S. real estate to a universe of global financial assets under an active strategy were mixed, although generally favorable for the highly risk-averse strategies; (4) correcting for second-moment smoothing in the real estate returns series had a relatively small impact for the more risk-tolerant strategies; and (5) there was some evidence that desmoothing resulted in improved probability estimates.

In several previous studies, discrete-time dynamic portfolio theory[1] was applied to the asset allocation problem, in conjunction with the empirical probability assessment approach (EPAA), to implement a set of active investment strategies. In the domestic setting (Grauer and Hakansson 1982, 1985 and

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[1] See Mossin (1968); Hakansson (1971, 1974); Leland (1972); Ross (1974); and Huberman and Ross (1983).

1986) the model was employed to construct and rebalance portfolios composed of U.S. stocks, corporate bonds, government bonds, and a riskfree asset. Borrowing was ruled out in the first article, while margin purchases were permitted in the other two. The third article also included small stocks as a separate investment vehicle. The probability distributions were naively estimated from past realized returns in the Ibbotson and Sinquefield 1926–1984 data base, and both annual and quarterly holding periods were employed from the mid-thirties forward. The results revealed that the gains from active diversification among the major asset categories were substantial, especially for the highly risk-averse strategies. In some cases, the returns from the active strategies were significantly higher than for fixed-weight rebalancing policies of similar riskiness.

In Grauer and Hakansson (1987) the model was applied to a global environment by including in the universe the four principal U.S. asset categories and up to fourteen non-U.S. equity and bond categories. The results showed that: (1) the gains from including non-U.S. asset classes in the universe were surprisingly large (in some cases statistically significant), especially for the highly risk-averse strategies; (2) the gains from removing the no leverage constraint were more substantial than they were in the absence of non-U.S. securities; and (3) there was strong evidence of market segmentation in that the optimal levels of investment in U.S. securities were mostly zero in the presence of the non-U.S. asset categories.

Finally, Grauer, Hakansson and Shen (1990) examined the asset allocation problem when the universe of risky assets was composed of twelve equal- and value-weighted industry components of the U.S. stock market. The results indicated that the active strategies of the dynamic investment model performed moderately well when applied to the value-weighted industry indices. Furthermore, the majority of the power policies generated statistically significant positive abnormal returns when managing the equal-weighted industry indices over both the full 1934–1986 period and the 1966–1986 subperiod. In the latter time frame, with one exception, the abnormal returns averaged two-thirds of total excess returns.

This paper compares the investment policies and returns for portfolios of stocks and bonds, with and without real estate. Besides long-term government bonds, long-term corporate bonds, stocks (the S&P 500 Index), and one-year Treasury Bills, three separate real estate asset classes were included: residential, farm and business real estate. Thus, a total of seven investment categories were available in the domestic setting. In the global case, the stock markets of Great Britain, Canada, France, Germany, Japan, the Netherlands and Switzerland were added, bringing the number of investment outlets to fourteen. Portfolios were revised annually over the period 1955–1988 in the domestic case and over the period 1970–1988 in the global case. This is a longer period than in previous studies involving real estate. In some runs, collaterized borrowing (up to posted margin requirements for financial assets and up to 80% for real estate) was permitted at the call money rate +1%.[2]

The next section outlines the discrete-time dynamic investment model and the method employed to make it operational. The model inputs for the unadjusted and desmoothed versions are described in the third and fourth sections and the fifth section identifies the data used. The following two sections report the results and statistical tests, while the final section contains a summary and concluding comments.

The Dynamic Investment Model

The basic discrete-time dynamic investment model used in this paper is the same as the one employed in Grauer and Hakansson (1986) and the reader is referred to that paper (see pp. 288–291) for details. It is based on the pure reinvestment version of discrete-time dynamic investment theory.[3] In particular, if $U_n(w_n)$ is the *induced* utility of wealth w_n with n periods to go (to the horizon) and r is the single-period return, the important convergence result (see Hakansson 1974),

$$U_n(w) \rightarrow \frac{1}{\gamma} w^{\gamma}, \quad \text{for some } \gamma < 1,$$

holds for a very broad class of terminal utility functions $U_0(w_0)$ when returns are independent (but nonstationary) from period to period. Convergence implies that use of the stationary, myopic decision rule

$$\max E\left[\frac{1}{\gamma}(1 + r)^{\gamma}\right], \quad \text{for some } \gamma < 1, \tag{1}$$

in each period is optimal. Consequently, the family of decision rules in equation (1) and encompasses a broad variety of different goal formulations for investors with intermediate- to long-term investment horizons.[4] Since the rel-

[2] While real estate borrowing rates are sensitive to the level of debt employed and other factors, a uniform borrowing rate was assumed for simplicity.

[3] The simple reinvestment formulation does ignore consumption of course.

[4] A plot of the functions $1/\gamma(1 + r)^{\gamma}$ for several values of γ was given in Grauer and Hakansson (1982, p. 42).

ative risk aversion function $(-wU''(w)/U'(w))$ for equation (1) is $1 - \gamma$, equation (1) incorporates the full range of risk attitudes from zero to infinity.

More specifically, at the beginning of each period t, the investor chooses a portfolio, x_t, on the basis of some member, γ, of the family of objective functions for returns r given by

$$V(1 + r) = \frac{1}{\gamma}(1 + r)^{\gamma}.$$

This is equivalent to solving the following problem in each period t:

$$\max_{x_t} E\left[\frac{1}{\gamma}(1 + r_t(x_t))^{\gamma}\right] = \max_{x_t} \sum_s \pi_{ts} \frac{1}{\gamma}(1 + r_{ts}(x_t))^{\gamma} \tag{2}$$

subject to:

$$x_{it} \geq 0, \quad x_{Lt} \geq 0, \quad x_{Bt} \leq 0, \quad \text{all } i, \tag{3}$$

$$\sum_i x_{it} + x_{Lt} + x_{Bt} = 1, \tag{4}$$

$$\sum_i m_{it} x_{it} \leq 1, \tag{5}$$

$$Pr\{1 + r_t(x_t) \geq 0\} = 1, \tag{6}$$

where

$r_{ts}(x_t) = \sum_i x_{it} r_{its} + x_{Lt} r_{Lt} + x_{Bt} r_{Bt}^d$ is the (ex ante) return on the portfolio in period t if state s occurs;

$\gamma \leq 1$ = a parameter that remains fixed over time;

x_{it} = the amount invested in risky asset category i in period t as a fraction of own capital;

x_{Lt} = amount lent in period t as a fraction of own capital;

x_{Bt} = amount borrowed in period t as a fraction of own capital;

x_t = $(x_{1t}, \ldots, x_{nt}, x_{Lt}, x_{Bt})$;

r_{it} = anticipated total return (dividend yield plus capital gains or losses) on asset category i in period t;

r_{Lt} = the return on the riskfree asset in period t;

r_{Bt}^d = interest rate on borrowing at the time of the decision at the beginning of period t;

m_{it} = initial margin requirement for asset category i in period t expressed as a fraction; and

π_{ts} = probability of state s at the end of period t, in which case the random return r_{it} will assume the value r_{its}.

Constraint in equation (3) rules out short sales[5] and in equation (4) is the budget constraint. Constraint in equation (5) serves to limit borrowing (when desired) to the maximum permissible under the margin requirements that apply to the various asset categories. Finally, constraint in equation (6) rules out any (ex ante) probability of bankruptcy.[6]

Model Inputs—Unadjusted Version

The inputs to the unadjusted model are based on the empirical probability assessment approach (EPAA). Consider the EPAA and suppose annual revision is used. At the beginning of year t, the portfolio problem, equations (2) through (6), use the following: the (observable) risk free return for year t, the (observable) call money rate of $+1\%$ in effect at the beginning of year t and the (observable) realized returns for the risky asset categories for the previous k years. Each *joint* realization in years $t - k$ through $t - 1$ is given probability $1/k$ of occurring in year t. Thus, under the EPAA, estimates are obtained on a moving basis and used in raw form, without adjustment. On the other hand, since the whole joint distribution is specified and used, there is no information loss; *all* moments and correlations are implicitly taken into account. It may be noted that the empirical distribution of the past k periods is optimal if the investor has no information about the form and parameters of the true distribution, but believes that this distribution went into effect k periods ago.[7]

With these inputs in place, the portfolio weights x_t for the various asset categories and the proportion of assets borrowed are calculated by solving equations (2) through (6) via nonlinear programming methods.[8] At the end of year

[5] Since short sales of even financial assets was not permissible in most countries during the period studied, they were ruled out in the interest of simplicity.

[6] The solvency constraint in equation (6) is not binding for the power functions, with $\gamma < 1$, and discrete probability distributions with a finite number of outcomes because the marginal utility of zero wealth is infinite. Nonetheless, it is convenient to explicitly consider equation (6) so that the nonlinear programming algorithm used to solve the investment problems does not attempt to evaluate an infeasible policy as it searches for the optimum.

[7] See Bawa, Brown and Klein (1979, p. 100).

[8] The nonlinear programming algorithm employed is described in Best (1975).

t, the realized returns on the risky assets are observed, along with the realized borrowing rate r^r_{Bt} (which may differ from the decision borrowing rate r^d_{Bt}).[9] Then, using the weights selected at the beginning of the year, the realized return on the portfolio chosen for year t is recorded. The cycle is then repeated in all subsequent years.[10]

All reported returns are gross of transaction costs and taxes and assume that the investor in question had no influence on prices. There are several reasons for this approach. First, as in our previous studies, we wish to keep the complications to a minimum. Second, the return series used as inputs and for comparisons also exclude transaction costs (for reinvestment of interest, dividends and real estate income) and taxes. Third, many investors are tax-exempt and techniques are available for keeping transaction costs low. Finally, since the proper treatment of these items is nontrivial, they are better left to a later study.

Data

The data used to estimate the probabilities of next period's returns on the risky assets, and to calculate their realized returns in each period, came from several sources. The annual returns series for the U.S. financial asset categories were obtained from Ibbotson Associates (1989). Real estate returns were obtained from Ibbotson and Siegel (1984) for the period 1947–1959, from Ibbotson, Siegel and Love (1985) for the years 1960–1984, and from Ibbotson Associates and Frank Russell Company for the 1985–1988 period.[11] The data base on non-U.S. equity returns for 1960–1988, covering seven countries (Canada, France, Germany, Japan, the Netherlands, Switzerland and the United Kingdom) was obtained from First Chicago Investment Advisors. All returns are expressed in U.S. dollars and represent total returns since both dividends (net of foreign taxes withheld) and capital appreciation or depreciation are taken into account.

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[9] The realized borrowing rate was calculated as a monthly average and was unknown at the beginning of the year when the portfolio was chosen.

[10] Note that if $k = 10$ under annual revision, then the first year for which a portfolio can be selected is $b + 10$, where b is the first year for which data is available.

[11] It is well known that data on real estate returns are less accurate than for financial asset returns since the latter are always based on transactions (or bids) while the former are generally based on a mixture of transactions and appraisals. The end result is that real estate returns may reflect some "smoothing." Furthermore, it is well known that parcels of real estate are to some extent unique. Therefore, as a practical matter it may be harder to match the returns on real estate indices than on indices of stocks or bonds. Nevertheless, we are compelled to work with what is available; the smoothing problem will be addressed in the next two sections.

The risk free asset was assumed to be 360-day U.S. Treasury Bills or U.S. Treasury Bonds maturing at the end of the year. We used the *Survey of Current Business* and *The Wall Street Journal*, as sources. The borrowing rate was assumed to be the call money rate +1% for *decision* purposes (but not for rate of return calculations). The applicable beginning of period decision rate, r^d_{Bt}, was viewed as persisting throughout the period and thus risk free. For 1955–1976, the call money rates were obtained from the *Survey of Current Business*; for later periods *The Wall Street Journal* was the source. Finally, margin requirements for stocks were obtained from the *Federal Reserve Bulletin*. Initial margins were set at 10% for government bonds, at 35% for corporate bonds, and at 20% for real estate. These levels are on the conservative side and are designed to compensate for the absence of maintenance requirements.[12]

Model Inputs—Adjusted Version

Extant data on real estate returns suffer from a number of shortcomings, when compared to returns series for stocks and bonds, as a number of observers have noted. In particular, stocks and at least some bonds trade in continuous auction markets characterized by large volume, many informed traders and low transaction costs. The general absence of these factors in real estate transactions and the necessary reliance on appraisals are viewed as resulting in *smoothed* returns data, understating the risks associated with real estate investment. Ross and Zisler (1991) estimate that real estate risk, as measured by the standard deviation of annual returns, lies somewhere "between that of stocks and bonds, in the 9% to 13% range." A similar finding was obtained by Geltner (1991). Most corrections for smoothing focus on the second moments.

There are at least two obstacles to properly *desmoothing* the real estate returns data in the present study. First, the discrete-time dynamic investment model does not give rise to (is not consistent with) a mean-variance formulation. Instead, the induced objective in equation (1) takes into account all the moments and co-moments of the joint return distribution for each period's decision. Thus, the advantage of being able to limit the adjustment to the second moment alone, is lost.

Second, while a mean-variance approximation of the objective function in equation (2) is possible. Such an approximation works well only under fre-

[12] There was no practical way to take maintenance margins into account in our programs. In any case, it is evident from the results that they would come into play only for the more risk-tolerant strategies, and even for them only occasionally, and that the net effect would be relatively neutral.

quent portfolio revision (Samuelson 1970; Ohlson 1975). Good results have also been obtained under quarterly rebalancing, but under annual revision, the approximation is rather poor for the high, such risk-averse strategies (see Grauer and Hakansson 1993a). Despite these provisos, we decided to proceed in order to have a means of desmoothing the real estate data series.

Let μ_{it} be the expected rate of return on asset category i in period t, and σ_{ijt} be the covariance between the returns on asset categories i and j in period t. The investment problem then becomes

$$\max_{x_t} \left\{ T(1 + \mu_t(x_t)) - \frac{1}{2} \sigma_t^2(x_t) \right\} \tag{7}$$

subject to equations (3) through (5), where the mean and variance of the portfolio are

$$\mu_t(x_t) = \sum_i x_{it}\mu_{it} + x_{Lt} r_{Lt} + x_{Bt} r_{Bt}^d,$$

$$\sigma_t^2(x_t) = \sum_i \sum_j x_{it} x_{jt} \sigma_{ijt},$$

and the risk tolerance parameter T is given by

$$T = 1/(1 - \gamma). \tag{8}$$

Despite the limitations noted above, the mean-variance approximation model was run for all values of T corresponding to the γ's employed in equation (2), with and without unsmoothed real estate variances, for the purpose of comparing the impact of desmoothing on both returns and optimal policies.[13] Desmoothing was accomplished by increasing the variance of the category s real estate returns over the estimating period $t - k$ to $t - 1$ by a factor $f_s > 1$.

Results—Unadjusted Version

Because of space limitations, only a portion of the results can be reported here. However, Figures 1 through 10 and Tables 2 through 11 provide a fairly representative sample of our findings. In most comparisons, we have calculated and included the returns on unlevered and levered equal-weighted benchmark portfolios of the risky assets. The compositions of these portfolios, along with

[13] For an analysis of estimation risk concerning the mean, see Grauer and Hakansson (1993b).

Table 1 ■ Asset category and fixed-weight portfolio symbols.

RL	Riskfree Lending	EW	Equal-weighted portfolio of risky assets in investment universe
B	Borrowing		
GB	Long-term U.S. Government Bonds	E2	20% in EW, 80% in RL
CB	Long-term U.S. Corporate Bonds	E4	40% in EW, 60% in RL
CS	U.S. Common Stocks (S & P 500)	E6	60% in EW, 40% in RL
FA	Farm Real Estate	E8	80% in EW, 20% in RL
RR	Residential Real Estate	E12	120% in EW, 20% in B
BR	Business Real Estate	E14	140% in EW, 40% in B
CA	Canadian Equities	E16	160% in EW, 60% in B
FR	French Equities	E18	180% in EW, 80% in B
GE	German Equities	E20	200% in EW, 100% in B
JA	Japanese Equities	IN	U.S. Inflation
NE	Dutch Equities		
SI	Swiss Equities		
UK	British Equities		

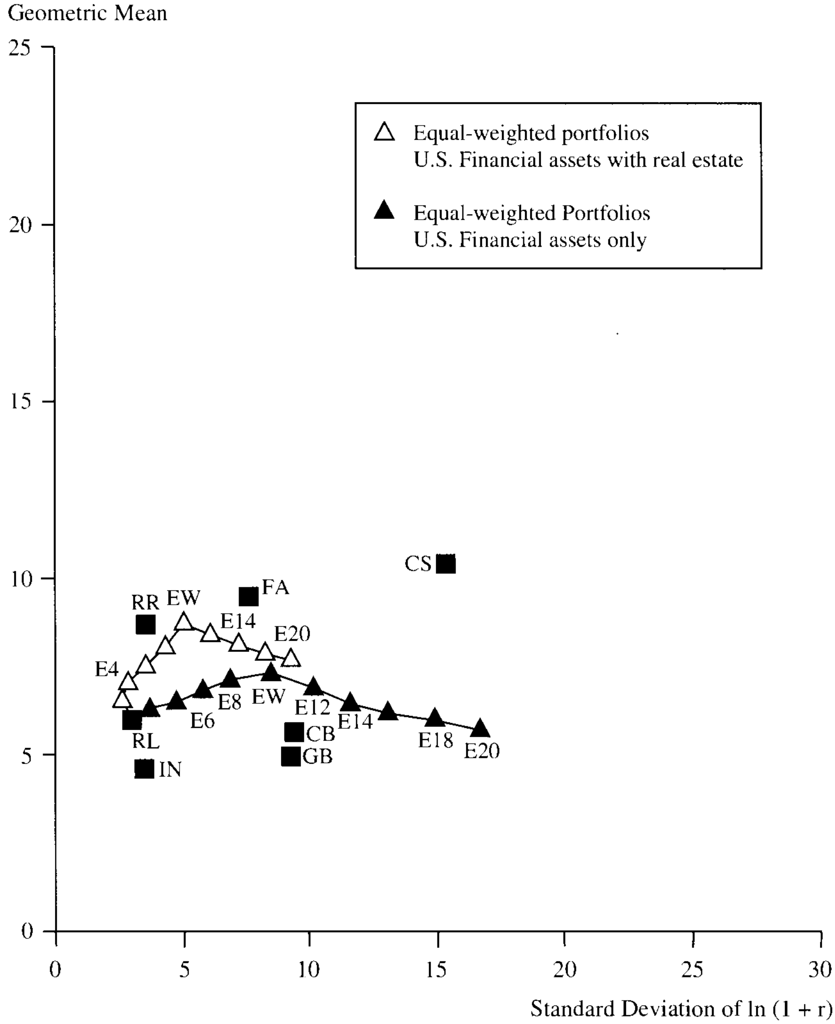
an enumeration of the asset categories included in the study, are summarized in Table 1.

Equal-weighted Portfolios

Figure 1 shows the geometric means and the standard deviations of the annual returns for 1955–1988 for the up- and down-levered equal-weighted portfolios with and without the real estate categories in the U.S. only case. Each portfolio was rebalanced at the beginning of every year.[14] When the two real estate categories were included, the annual geometric means increased by about 1%, while the standard deviations were halved. Thus, reduced total risk was the

[14] Equal-weighted portfolios are often referred to as semipassive since rebalancing is typically required at the beginning of each period. Value-weighted portfolios, on the other hand, are *fully* passive since, in the absence of company recapitalizations, only dividends and/or interest need to be reinvested.

Figure 1 ■ Geometric means and standard deviations of annual returns. U.S. Financial assets with and without real estate, equal-weighted portfolios, 1955–1988.



RL	Riskfree Lending	EW	Equal-weighted portfolio of risky assets in investment universe
GB	Long-term U.S. Government Bonds		
CB	Long-term U.S. Corporate Bonds	E4	40% in EW, 60% in RL
CS	U.S. Common Stocks (S & P 500)	E6	60% in EW, 40% in RL
FA	Farm Real Estate	E8	80% in EW, 20% in RL
RR	Residential Real Estate	E12	120% in EW, 20% in B
IN	U.S. Inflation	E14	140% in EW, 40% in B
		E18	180% in EW, 80% in B
		E20	200% in EW, 100% in B

predominant impact from adding real estate to an equal-weighted portfolio of domestic financial assets.

Similarly, Figure 6 shows the geometric means and the standard deviations for the 1970–1988 period for the equal-weighted portfolios with and without the three real estate categories, in the global case. In contrast to the domestic case, the impact on the geometric means is negligible and the reduction in total risk rather small.

U.S. Universe—No Leverage Case

Annual revision strategies were run: (1) based on a six asset category universe (risk free lending, U.S. governments, U.S. corporates, U.S. equities, farm real estate and residential real estate) over the full 1955–1988 period; (2) for the previous categories plus business real estate (for which data go back only to 1960) over the shorter 1968–1988 subperiod; and (3) for the four U.S. financial asset classes only. The left side of Table 2 shows, and Figure 2 plots, the annual geometric means and the standard deviations of the realized returns for each of the six asset components (see squares), for the semi-passive equal-weighted portfolios including real estate (see triangles), as well as for ten active strategies corresponding to γ's in (1) ranging from -50 (extremely risk averse) to 1, under annual revision, for the 34-year period 1955–1988, both with and without the real estate categories (see black circles vs. white circles). The estimating period was eight years. Among the asset categories, U.S. equities had both the highest geometric mean (10.4%) and highest volatility (15.3%). Farm real estate achieved the second highest geometric mean return (9.4%), with much smaller variability (7.6%).

As both Figure 2 and the left side of Table 2 show, the presence of farm and residential real estate as investment opportunities tended to increase the annual geometric mean returns for the active strategies by about 300 basis points per year, with very little change in the standard deviations of returns.[15]

The right side of Table 2 and Figure 3 show the geometric means and standard deviations of the returns for the 1968–1988 subperiod, both with and without farm and residential real estate. Figure 3 also shows the results when business real estate was added. Over the latter subperiod, there was little impact on standard deviations from the availability of real estate, but a rather dramatic

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[15] For consistency with the geometric mean, the standard deviation is based on the log of 1 plus the rate of return. This quantity is very similar to the standard deviation of the rate of return for levels less than 25%.

Table 2 ■ Geometric means and standard deviations of annual returns. U.S. Financial assets with and without real estate, 1955–1988 and 1968–1988. (Annual portfolio revision, borrowing precluded, 8-year estimating period.)

	1955–1988		1968–1988	
Portfolio	Geometric Mean	Standard Deviation*	Geometric Mean	Standard Deviation*
Common Stocks	10.4	15.3	9.6	15.7
Long-term Government Bonds	4.9	9.6	7.4	10.7
Long-term Corporate Bonds	5.5	9.7	8.0	11.2
Residential Real Estate	8.1	3.4	10.0	3.3
Farm Real Estate	9.4	7.6	9.8	9.5
Business Real Estate			10.1	3.0
Lending	6.1	2.9	7.8	2.5
Inflation	4.5	3.2	6.2	3.1
Portfolio E2	6.5	2.8	8.1	2.4
Portfolio E4	6.9	3.0	8.5	2.9
Portfolio E6	7.3	3.5	8.8	3.7
Portfolio E8	7.7	4.2	9.1	4.7
Portfolio EW	8.0	5.0	9.4	5.7
U.S. Financial Assets Only				
Power −50	6.6	3.7	7.9	2.5
Power −30	6.9	4.2	8.0	2.6
Power −15	7.5	6.1	8.2	3.1
Power −10	8.0	7.0	8.4	3.7
Power −5	9.0	9.1	8.9	5.2
Power −3	9.6	10.3	9.1	6.7
Power −1	10.1	11.7	9.1	9.3
Power 0	10.1	11.9	9.1	9.6
Power 0.5	10.1	11.9	9.1	9.6
Power 1	10.1	11.9	9.1	9.6

Table 2 ■ (continued)

Portfolio	1955–1988 Geometric Mean	1955–1988 Standard Deviation*	1968–1988 Geometric Mean	1968–1988 Standard Deviation*
U.S. Financial Assets Plus Real Estate				
Power −50	10.3	4.6	12.1	4.8
Power −30	10.7	5.4	12.3	5.8
Power −15	11.0	6.7	12.7	6.4
Power −10	11.0	7.4	12.6	6.7
Power −5	11.3	9.0	12.6	7.3
Power −3	11.5	10.4	12.8	7.5
Power −1	12.2	10.9	13.2	7.9
Power 0	12.3	11.4	13.2	8.8
Power 0.5	12.7	11.7	13.7	9.3
Power 1	12.9	11.7	13.7	9.3

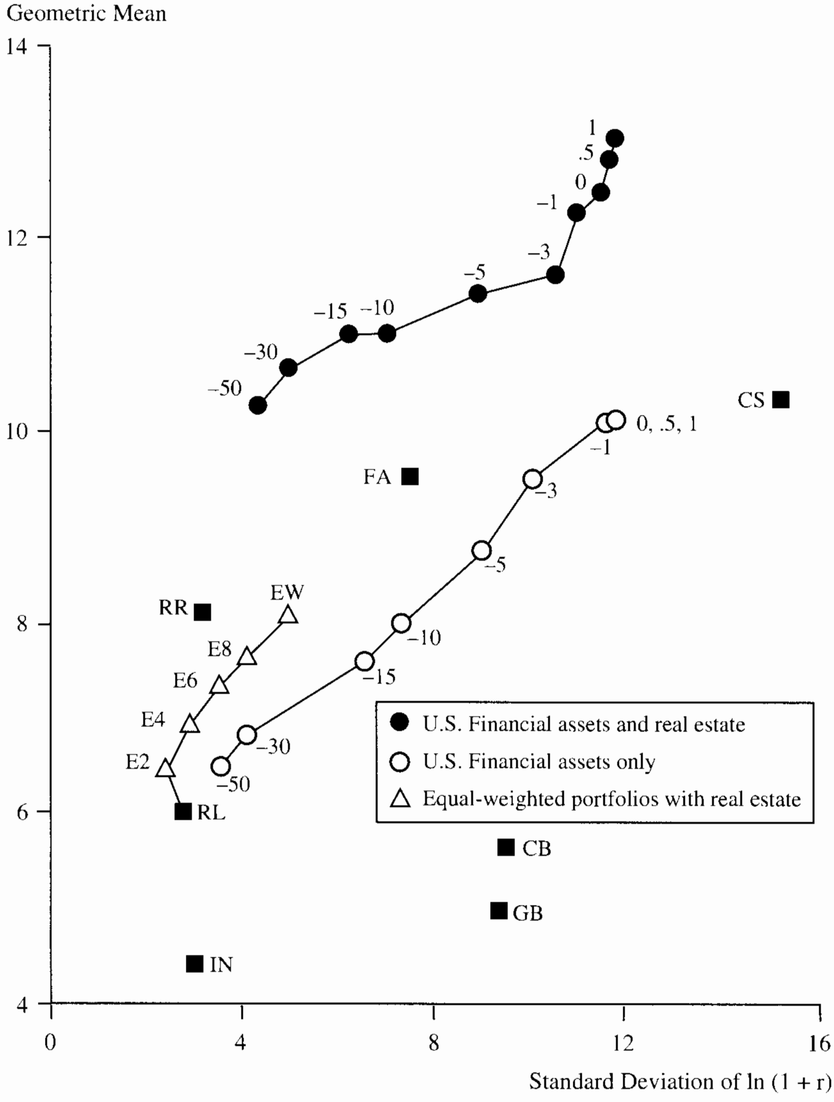
* Standard deviation of $\ln(1 + r)$

increase in the annual geometric means (approximately 400 basis points). Surprisingly, the addition of business real estate to the other categories (see white squares) had very little impact on either the geometric means or the standard deviations.[16]

Table 3 shows the portfolio compositions and the year-by-year returns for the power −50 strategy in the no-leverage case, both when farm real estate and residential real estate were included in the universe and when they were not. In the financial assets only case, holdings were concentrated in the risk free asset, with a scattering of small positions in stocks, government bonds and corporate bonds. Losses were incurred in two years out of the 34. When the real estate asset categories were included, however, we see a very different pattern. In most years, large positions in residential and farm real estate eliminated all holdings in the riskfree asset. These real estate categories also prac-

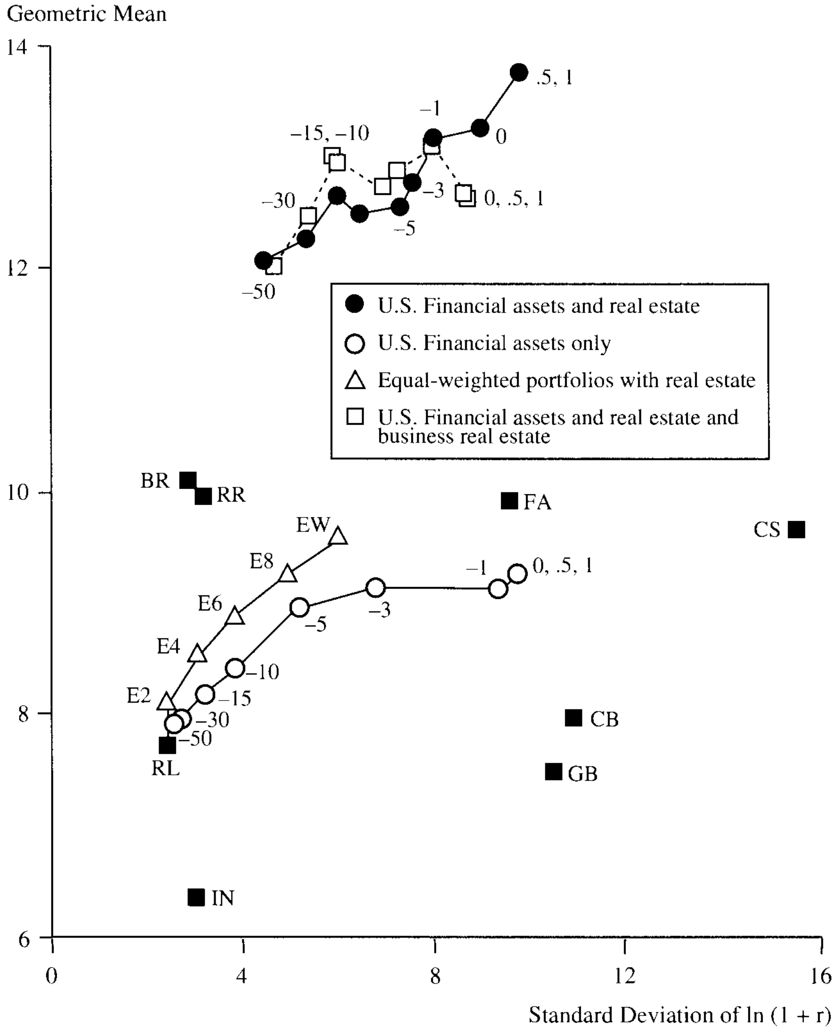
[16] Recall that returns data for business real estate were not available before 1960.

Figure 2 ■ Geometric means and standard deviations of annual returns. U.S. Financial assets with and without real estate, 1955–1988. (Annual portfolio revision, borrowing precluded, 8-year estimating period.)



RL	Riskfree Lending		
GB	Long-term U.S. Government Bonds	EW	Equal-weighted portfolio of risky assets in investment universe
CB	Long-term U.S. Corporate Bonds		
CS	U.S. Common Stocks (S & P 500)	E2	20% in EW, 80% in RL
FA	Farm Real Estate	E4	40% in EW, 60% in RL
RR	Residential Real Estate	E6	60% in EW, 40% in RL
IN	U.S. Inflation	E8	80% in EW, 20% in RL

Figure 3 ■ Geometric means and standard deviations of annual returns. U.S. Financial assets with and without real estate, 1968–1988. (Annual portfolio revision, borrowing precluded, 8-year estimating period.)



RL	Riskfree Lending		
GB	Long-term U.S. Government Bonds	EW	Equal-weighted portfolio of risky assets in investment universe
CB	Long-term U.S. Corporate Bonds		
CS	U.S. Common Stocks (S & P 500)	E2	20% in EW, 80% in RL
FA	Farm Real Estate	E4	40% in EW, 60% in RL
RR	Residential Real Estate	E6	60% in EW, 40% in RL
BR	Business Real Estate	E8	80% in EW, 20% in RL
IN	U.S. Inflation		

Table 3 ■ Portfolio compositions and realized returns for power −50. U.S. Financial assets with and without real estate, 1955–1988. (Annual revision, borrowing precluded, 8-year estimating period.)

	U.S. Financial Assets Only Investment Fractions						U.S. Financial Assets Plus Real Estate Investment Fractions							
Period	Return	RL	B	CS	GB	CB	Return	RL	B	CS	GB	CB	RR	FA
1955	18.3			0.6		0.4	10.1			0.2			0.8	
1956	−2.1			0.4		0.7	4.7			0.3			0.7	
1957	−0.7	0.7		0.3			1.4			0.3			0.7	
1958	7.4	0.8		0.1		0.1	9.4			0.1			0.7	0.2
1959	3.9	0.9		0.1			4.6			0.1			0.7	0.2
1960	4.3	0.9		0.1			4.2	0.6		0.1				0.3
1961	5.3	0.8		0.1	0.1		8.0			0.1			0.7	0.2
1962	1.9	0.8		0.1	0.1		7.3			0.1	0.1			0.9
1963	4.8	0.8		0.1		0.1	10.3			0.1				0.9
1964	4.8	0.9		0.1			7.6							1.0
1965	3.1	0.2		0.2		0.6	12.0			0.1	0.1			0.9
1966	2.7	0.8		0.1		0.1	8.7			0.1	0.1			0.8
1967	6.0	1.0		0.1			9.9			0.1	0.1			0.8
1968	6.3	1.0		0.1			7.7			0.1	0.1			0.8
1969	5.7	0.9		0.1			6.3			0.1				0.9
1970	8.2	1.0					7.8							1.0
1971	5.5	1.0		0.1			11.3						0.1	0.9
1972	5.0	0.9		0.1			17.1						0.2	0.9
1973	5.0	1.0					28.7						0.2	0.8
1974	7.3	1.0					18.0						0.3	0.7
1975	7.1	1.0					16.4						0.4	0.6
1976	6.1	1.0					13.2						0.5	0.5
1977	4.4	0.9			0.1		11.3						0.5	0.5
1978	6.5	0.9			0.1		15.5					0.1	0.2	0.7
1979	10.5	1.0					20.9							1.0
1980	11.3	1.0					12.6							1.0
1981	13.8	1.0					7.0	0.4						0.6
1982	13.3	1.0					11.4	0.9						0.1

Table 3 ■ (continued)

	U.S. Financial Assets Only Investment Fractions						U.S. Financial Assets Plus Real Estate Investment Fractions							
Period	Return	RL	B	CS	GB	CB	Return	RL	B	CS	GB	CB	RR	FA
1983	9.1	0.9		0.1			7.4	0.5						0.5
1984	9.9	1.0		0.1			9.3	0.8						0.1
1985	10.0	1.0					8.6	0.8						0.2
1986	9.0	0.9		0.1			9.3	0.7			0.1			0.2
1987	5.3	0.8		0.1	0.1		8.8				0.1		0.9	
1988	7.5	0.8		0.1			7.9				0.1		0.9	

RL Riskfree Lending
B Borrowing
CS U.S. Common Stocks (S & P 500)
GB Long-term U.S. Government Bonds
CB Long-term U.S. Corporate Bonds
RR Residential Real Estate
FA Farm Real Estate

tically replaced all positions in corporate bonds. A positive return was earned in each year.

Table 4, which shows the portfolio holdings and annual returns for the power 0 (growth-optimal) strategy, is perhaps even more striking. In the financial assets only case, asset holdings were never diversified, but alternated (remarkably infrequently) between common stocks, lending and corporate bonds. With the real estate categories included, some diversification took place, but the principal difference was the replacement of all lending and some equity investments by positions in farm real estate. The portfolio changes were infrequent, which presumably resulted in low overall transactions costs. Note also, however, that this investor was in real estate less than half of the time (16 years out of 34).

U.S. Universe—Leverage Case

When the borrowing constraint is removed in the previous analysis, Table 2 is replaced by Table 5 and Figures 2 and 3 by Figures 4 and 5.

Table 4 ■ Portfolio compositions and realized returns for power 0. U.S. Financial assets with and without real Estate, 1955–1988. (Annual revision, borrowing precluded, 8-year estimating period.)

	U.S. Financial Assets Only Investment Fractions						U.S. Financial Assets Plus Real Estate Investment Fractions							
Period	Return	RL	B	CS	GB	CB	Return	RL	B	CS	GB	CB	RR	FA
1955	31.6			1.0			31.6			1.0				
1956	6.6			1.0			6.6			1.0				
1957	−10.8			1.0			−10.8			1.0				
1958	43.4			1.0			43.4			1.0				
1959	12.0			1.0			12.0			1.0				
1960	0.5			1.0			26.9			1.0				
1962	−8.7			1.0			−8.7			1.0				
1963	22.8			1.0			22.8			1.0				
1964	16.5			1.0			16.5			1.0				
1965	12.5			1.0			12.5			1.0				
1966	−10.1			1.0			−10.1			1.0				
1967	24.0			1.0			14.3			0.3				0.7
1968	11.1			1.0			10.3			0.7				0.3
1969	−8.5			1.0			−8.5			1.0				
1970	8.2	1.0					7.8							1.0
1971	14.3			1.0			11.7							1.0
1972	19.0			1.0			18.9							1.0
1973	−14.7			1.0			35.3							1.0
1974	7.3	1.0					19.8							1.0
1975	7.1	1.0					18.5							1.0
1976	6.1	1.0					19.5							1.0
1977	1.7					1.0	11.2							1.0
1978	−0.1					1.0	18.2							1.0
1979	10.5	1.0					20.9							1.0
1980	11.3	1.0					12.6							1.0
1981	13.8	1.0					2.2							1.0

Table 4 ■ (continued)

	U.S. Financial Assets Only Investment Fractions						U.S. Financial Assets Plus Real Estate Investment Fractions							
Period	Return	RL	B	CS	GB	CB	Return	RL	B	CS	GB	CB	RR	FA
1982	13.3	1.0					3.1							1.0
1983	22.5			1.0			22.5			1.0				
1984	6.3			1.0			6.3			1.0				
1985	32.2			1.0			21.4			0.6			0.4	
1986	18.5			1.0			18.5			1.0				
1987	5.2			1.0			5.2			1.0				
1988	16.8			1.0			16.8			1.0				

RL	Riskfree Lending
B	Borrowing
CS	U.S. Common Stocks (S & P 500)
GB	Long-term U.S. Government Bonds
CB	Long-term U.S. Corporate Bonds
RR	Residential Real Estate
FA	Farm Real Estate

The use of leverage substantially increased the differences in geometric means between the financial assets only case and that in which real estate was a possible investment outlet. At the same time, the increase in standard deviation for each power was relatively modest. For power −3, for example, the availability of leverage increased the geometric mean, for the full 34-year period 1955–1988, from 9.5% in the financial assets only case to 21.6% with farm and residential real estate available as investment outlets, while the standard deviation increased only from 12.8% to 20.2%. Figure 5 shows that the inclusion of business real estate as an additional investment opportunity had a slightly negative effect. While this may seem somewhat surprising, it should be noted that only 21 consecutive portfolio decisions took place over the 1968–1988 subperiod.

Tables 6 and 7 show the portfolio choices and year-by-year returns when borrowing is permitted for powers −50 and 0. In the conservative power −50

Table 5 ■ Geometric means and standard deviations of annual returns. U.S. Financial assets with and without real estate, 1955–1988 and 1968–1988. (Annual portfolio revision, borrowing permitted, 8-year estimating period.)

Portfolio	1955–1988 Geometric Mean	1955–1988 Standard Deviation*	1968–1988 Geometric Mean	1968–1988 Standard Deviation*
Common Stocks	10.4	15.3	9.6	15.7
Long-term Government Bonds	4.9	9.6	7.4	10.7
Long-term Corporate Bonds	5.5	9.7	8.0	11.2
Residential Real Estate	8.1	3.4	10.0	3.3
Farm Real Estate	9.4	7.6	9.8	9.5
Business Real Estate			10.1	3.0
Lending	6.1	2.9	7.8	2.5
Inflation	4.5	3.2	6.2	3.1
Portfolio E2	6.5	2.8	8.1	2.4
Portfolio E4	6.9	3.0	8.5	2.9
Portfolio E6	7.3	3.5	8.8	3.7
Portfolio E8	7.7	4.2	9.1	4.7
Portfolio EW	8.0	5.0	9.4	5.7
Portfolio E12	8.0	6.0	9.2	7.0
Portfolio E14	7.9	6.9	9.0	8.1
Portfolio E16	7.8	7.7	8.9	9.3
Portfolio E18	7.8	8.6	8.7	10.5
Portfolio E20	7.6	9.5	8.5	11.7
U.S. Financial Assets Only				
Power −50	6.6	3.7	7.9	2.5
Power −30	6.9	4.2	8.0	2.6
Power −15	7.5	6.1	8.2	3.1
Power −10	8.1	7.8	8.4	3.7
Power −5	9.1	10.8	8.8	5.3
Power −3	9.5	12.8	9.0	7.2

Table 5 ■ (continued)

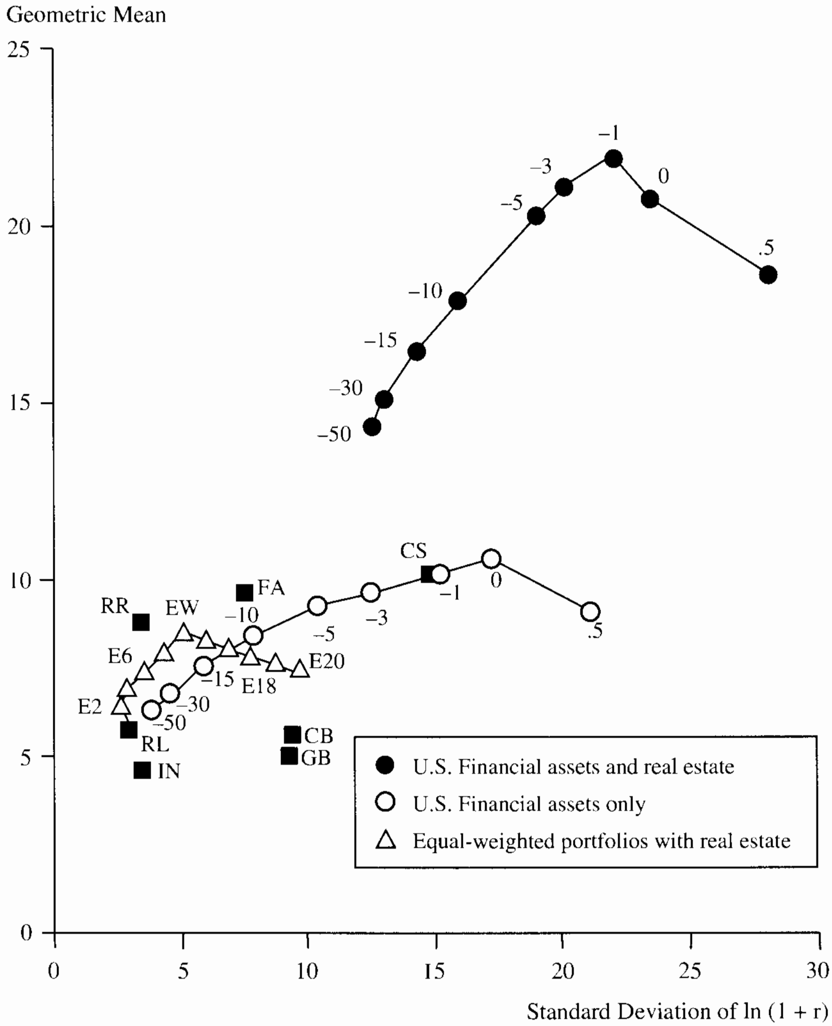
Portfolio	1955–1988 Geometric Mean	1955–1988 Standard Deviation*	1968–1988 Geometric Mean	1968–1988 Standard Deviation*
Power −1	10.5	15.7	9.0	11.0
Power 0	10.7	17.2	8.5	13.5
Power 0.5	9.3	22.0	6.2	21.8
Power 1	−2.2	64.4	−11.4	79.2
U.S. Financial Assets Plus Real Estate				
Power −50	14.2	12.6	18.5	14.8
Power −30	14.8	12.8	18.5	15.1
Power −15	16.6	14.2	19.6	16.4
Power −10	18.0	15.9	20.6	18.1
Power −5	20.7	19.3	23.3	21.6
Power −3	21.6	20.2	24.5	22.4
Power −1	22.3	22.1	26.9	23.8
Power 0	20.9	23.6	24.8	26.3
Power 0.5	17.8	28.9	20.1	34.1
Power 1	5.4	69.4	1.4	87.5

* Standard deviation of $\ln(1 + r)$

case, the investor placed funds in real estate each year and sharply reduced lending compared to the financial assets only case. In addition, to our great surprise, this investor occasionally borrowed heavily for the purpose of investing in residential and farm real estate—with surprisingly positive returns (82.8% vs. 4.9% in 1973, for example). Perhaps even more surprisingly, this investor experienced two loss years in the absence of real estate and only one loss year *with* real estate.

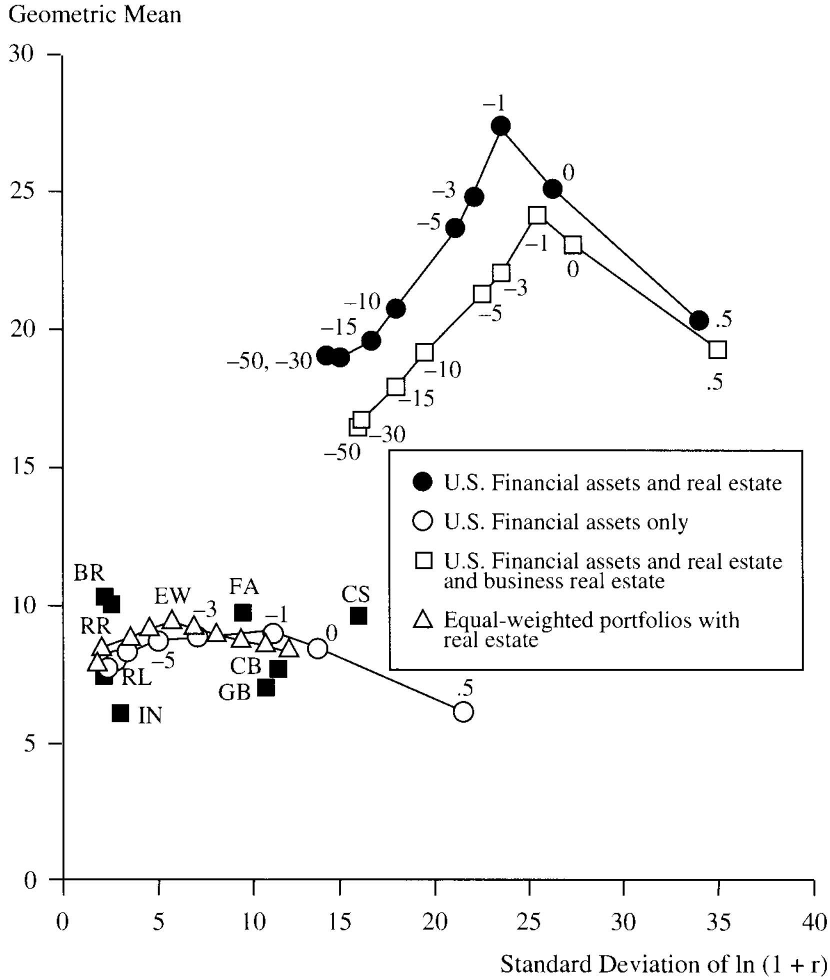
When real estate was available as an outlet, the more aggressive power 0 investor was a borrower every year except 1981 in the absence of real estate, the investor was a much more modest borrower, as Table 7 shows. In the latter

Figure 4 ■ Geometric means and standard deviations of annual returns. U.S. Financial assets with and without real estate, 1955–1988. (Annual portfolio revision, borrowing permitted, 8-year estimating period.)



RL	Riskfree Lending	EW	Equal-weighted portfolio of risky assets in investment universe
GB	Long-term U.S. Government Bonds		
CB	Long-term U.S. Corporate Bonds	E2	20% in EW, 80% in RL
CS	U.S. Common Stocks (S & P 500)	E6	60% in EW, 40% in RL
FA	Farm Real Estate	E18	180% in EW, 80% in B
RR	Residential Real Estate	E20	200% in EW, 100% in B
IN	U.S. Inflation		

Figure 5 ■ Geometric means and standard deviations of annual returns. U.S. Financial assets with and without real estate, 1968–1988. (Annual portfolio revision, borrowing permitted, 8-year estimating period.)



RL	Riskfree Lending	EW	Equal-weighted portfolio of risky assets in investment universe
GB	Long-term U.S. Government Bonds		
CB	Long-term U.S. Corporate Bonds		
CS	U.S. Common Stocks (S & P 500)		
FA	Farm Real Estate		
RR	Residential Real Estate		
BR	Business Real Estate		
IN	U.S. Inflation		

Table 6 ■ Portfolio compositions and realized returns for power −50. U.S. Financial assets with and without real estate, 1955–1988. (Annual revision, borrowing permitted, 8-year estimating period.)

	U.S. Financial Assets Only Investment Fractions						U.S. Financial Assets Plus Real Estate Investment Fractions							
Period	Return	RL	B	CS	GB	CB	Return	RL	B	CS	GB	CB	RR	FA
1955	18.3			0.6		0.4	10.1			0.2			0.8	
1956	−2.1			0.4		0.7	4.7			0.3			0.7	
1957	−0.7	0.7		0.3			1.4			0.3			0.7	
1958	7.4	0.8		0.1		0.1	9.4			0.1			0.7	0.2
1959	3.9	0.9		0.1			4.6			0.1			0.7	0.2
1960	4.3	0.9		0.1			4.2	0.6		0.1				0.3
1961	5.3	0.8		0.1	0.1		8.0			0.1			0.7	0.2
1962	1.9	0.8		0.1	0.1		7.3		0.1	0.1				0.9
1963	4.8	0.8		0.1		0.1	10.7			0.1				1.0
1964	4.8	0.9		0.1			7.6							1.0
1965	3.1	0.2		0.2		0.6	12.8		−0.2	0.1	0.1			1.0
1966	2.7	0.8		0.1		0.1	8.7			0.1	0.1			0.8
1967	6.0	1.0		0.1			9.9			0.1	0.1			0.8
1968	6.3	1.0		0.1			7.7			0.1	0.1			0.8
1969	5.7	0.9		0.1			−1.5		−3.5	0.2				4.3
1970	8.2	1.0					7.8							1.0
1971	5.5	1.0		0.1			25.0		−3.6	0.2			1.4	3.0
1972	5.0	0.9		0.1			61.3		−4.0				0.7	4.3
1973	5.0	1.0					82.8		−4.0				2.0	3.0
1974	7.3	1.0					18.0						0.3	0.7
1975	7.1	1.0					16.4						0.4	0.6
1976	6.1	1.0					31.4		−4.0				2.7	2.2
1977	4.4	0.9			0.1		28.2		−4.0				3.1	1.9
1978	6.5	0.9				0.1	37.1		−4.2		0.4		2.4	2.3
1979	10.5	1.0					20.9							1.0
1980	11.3	1.0					12.6							1.0
1981	13.8	1.0					7.0	0.4						0.6

Table 6 ■ (continued)

	U.S. Financial Assets Only Investment Fractions						U.S. Financial Assets Plus Real Estate Investment Fractions							
Period	Return	RL	B	CS	GB	CB	Return	RL	B	CS	GB	CB	RR	FA
1982	13.3	1.0					11.4	0.9						0.1
1983	9.1	0.9		0.1			7.4	0.5					0.5	
1984	9.9	1.0		0.1			9.3	0.8					0.1	
1985	10.0	1.0					8.6	0.8					0.2	
1986	9.0	0.9		0.1			9.3	0.7		0.1			0.2	
1987	5.3	0.8		0.1	0.1		8.8				0.1		0.9	
1988	7.5	0.8		0.1			7.9				0.1		0.9	

RL	Riskfree Lending
B	Borrowing
CS	U.S. Common Stocks (S & P 500)
GB	Long-term U.S. Government Bonds
CB	Long-term U.S. Corporate Bonds
RR	Residential Real Estate
FA	Farm Real Estate

case, this investor kept *all* money in riskfree lending eight years out of 34. Again, perhaps surprisingly, this investor lost money eight times in the financial assets only case but only five times when real estate was available for investing. Real estate investments were sharply curtailed after 1980.

Global Universe

Figure 7 and the left side of Table 8 show the annual geometric means and standard deviations of the portfolio returns with borrowing precluded for: (1) the semipassive equal-weighted portfolios including real estate (see triangles); (2) the active financial asset categories only case (risk free lending, U.S. corporate bonds, U.S. government bonds, and U.S., Canadian, Dutch, French, German, Japanese, Swiss and British equities—see white circles); and (3) for the same categories plus the three real estate outlets (business, farm, and res-

Table 7 ■ Portfolio compositions and realized returns for power 0. U.S. Financial assets with and without real estate, 1955–1988. (Annual revision, borrowing permitted, 8-year estimating period.)

	U.S. Financial Assets Only Investment Fractions						U.S. Financial Assets Plus Real Estate Investment Fractions							
Period	Return	RL	B	CS	GB	CB	Return	RL	B	CS	GB	CB	RR	FA
1955	49.8		−0.7	1.7			13.1		−4.0					5.0
1956	7.2		−0.4	1.4			11.2		−1.7	0.9				1.8
1957	−17.8		−0.4	1.4			−17.8		−0.4	1.4				
1958	60.0		−0.4	1.4			50.6		−2.7	0.5				3.1
1959	12.7		−0.1	1.1			6.6		−2.6	0.4				3.2
1960	−0.1		−0.1	1.1			−0.1		−0.1	1.1				
1961	36.1		−0.4	1.4			36.1		−0.4	1.4				
1962	−14.8		0.4	1.4			−14.8		−0.4	1.4				
1963	40.1		−1.0	2.0			31.9		−2.9	0.8				3.1
1964	21.2		−0.4	1.4			15.9		−4.0					5.0
1965	15.4		−0.4	1.4			40.4		−4.0					5.0
1966	−17.3		−0.4	1.4			15.5		−2.7	0.5				3.2
1967	26.5		−0.1	1.1			24.9		−4.0					5.0
1968	12.7		−0.4	1.4			14.2		−4.0					5.0
1969	−12.9		−0.3	1.3			0.4		−4.0					5.0
1970	8.2	1.0					3.1		−4.0					5.0
1971	15.9		−0.2	1.2			31.6		−4.0					5.0
1972	26.9		−0.6	1.6			70.1		−4.0					5.0
1973	−14.7			1.0			139.4		−4.0					5.0
1974	7.3	1.0					51.1		−4.0					5.0
1975	7.1	1.0					56.5		−4.0					5.0
1976	6.1	1.0					64.0		−4.0					5.0
1977	1.6					1.0	27.4		−4.0					5.0
1978	−1.7		−0.2			1.2	53.5		−4.0					5.0
1979	10.5	1.0					53.0		−4.0					5.0
1980	11.3	1.0					1.8		−4.0					5.0

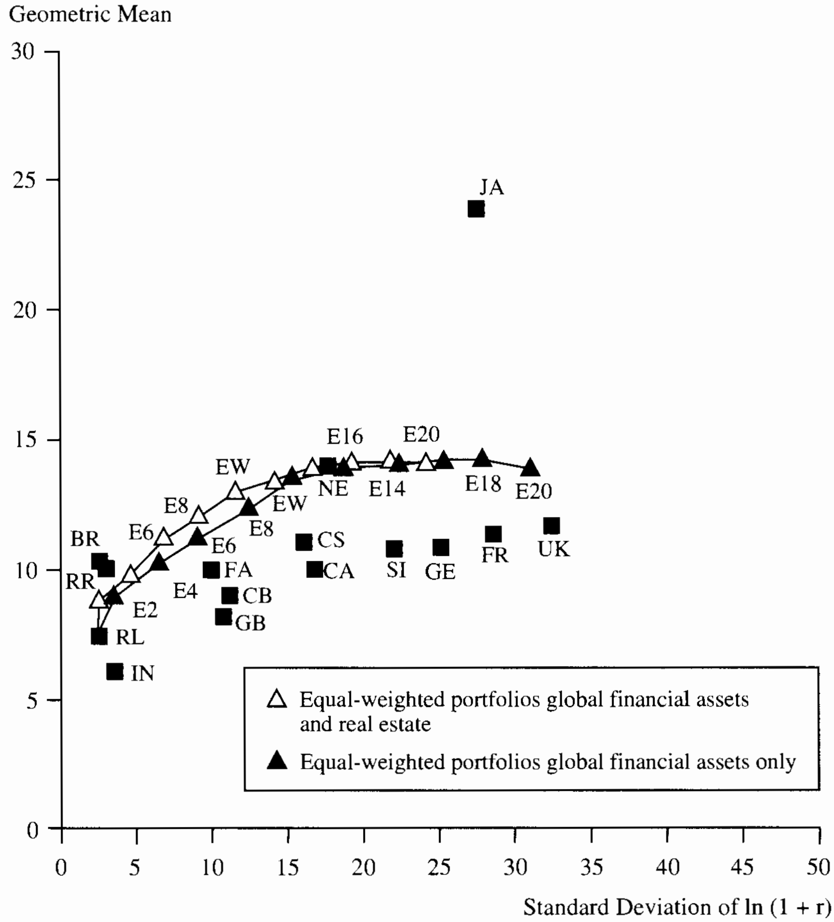
Table 7 ■ (continued)

	U.S. Financial Assets Only Investment Fractions						U.S. Financial Assets Plus Real Estate Investment Fractions							
Period	Return	RL	B	CS	GB	CB	Return	RL	B	CS	GB	CB	RR	FA
1981	13.8	1.0					2.2							1.0
1982	13.3	1.0					−16.7		−0.8					1.8
1983	23.0			1.0			23.0			1.0				
1984	4.8		−0.3	1.3			4.8		−0.3	1.3				
1985	34.0		−0.1	1.1			2.6		−3.4	0.4			4.0	
1986	28.2		−1.0	2.0			28.2		−1.0	2.0				
1987	−21.0		2.7	1.6	2.1		−21.0		−2.7	1.6	2.1			
1988	23.7		−2.5	1.6	1.9		23.7		−2.5	1.6	1.9			

RL Riskfree Lending
B Borrowing
CS U.S. Common Stocks (S & P 500)
GB Long-term U.S. Government Bonds
CB Long-term U.S. Corporate Bonds
RR Residential Real Estate
FA Farm Real Estate

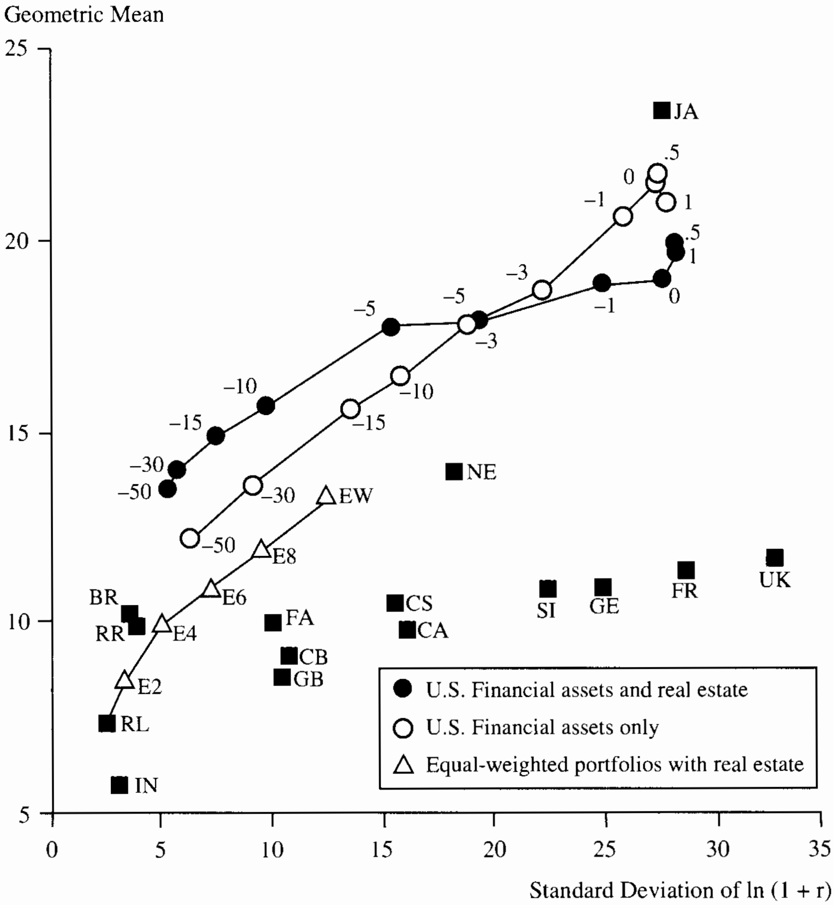
idential—see black circles). In this case, it is evident that the inclusion of U.S. real estate had relatively little impact on portfolios and returns. On the other hand, with borrowing permitted, the impact was more pronounced, with substantially improved returns for the highly risk-averse strategies and substantially diminished returns for the more risk-tolerant strategies (see Figure 8 and the right side of Table 8). As Table 9 shows (for the power −3 investor), farm real estate tended to replace the financial assets more than the other two real estate categories. Note also that the above investor, who attained an annual geometric mean return of 28.7%, consistently ignored only four asset categories: risk free lending, U.S. corporate bonds, and German and French equities. Furthermore, the investor experienced only two loss years out of 19 and stayed out of farm real estate entirely after 1984.

Figure 6 ■ Geometric means and standard deviations of annual returns. Global financial assets with and without real estate, equal-weighted portfolios, 1970–1988.



RL	Riskfree Lending	EW	Equal-weighted portfolio of risky assets in investment universe
GB	Long-term U.S. Government Bonds	E2	20% in EW, 80% in RL
CB	Long-term U.S. Corporate Bonds	E4	40% in EW, 60% in RL
CS	U.S. Common Stocks (S & P 500)	E6	60% in EW, 40% in RL
FA	Farm Real Estate	E8	80% in EW, 20% in RL
RR	Residential Real Estate	E14	140% in EW, 40% in B
BR	Business Real Estate	E16	160% in EW, 60% in B
CA	Canadian Equities	E18	180% in EW, 80% in B
FR	French Equities	E20	200% in EW, 100% in B
GE	German Equities		
JA	Japanese Equities		
NE	Dutch Equities		
SI	Swiss Equities		
UK	British Equities		
IN	U.S. Inflation		

Figure 7 ■ Geometric means and standard deviations of annual returns. Global financial assets with and without real estate, 1970–1988. (Annual portfolio revision, borrowing precluded, 10-year estimating period.)



RL	Riskfree Lending		
GB	Long-term U.S. Government Bonds	EW	Equal-weighted portfolio of risky assets in investment universe
CB	Long-term U.S. Corporate Bonds		
CS	U.S. Common Stocks (S & P 500)	E2	20% in EW, 80% in RL
FA	Farm Real Estate	E4	40% in EW, 60% in RL
RR	Residential Real Estate	E6	60% in EW, 40% in RL
BR	Business Real Estate	E8	80% in EW, 20% in RL
CA	Canadian Equities		
FR	French Equities		
GE	German Equities		
JA	Japanese Equities		
NE	Dutch Equities		
SI	Swiss Equities		
UK	British Equities		
IN	U.S. Inflation		

Table 8 ■ Geometric means and standard deviations of annual returns for U.S. financial assets, real estate, and global assets, 1970–88. (Annual portfolio revision, 10-year estimating period.)

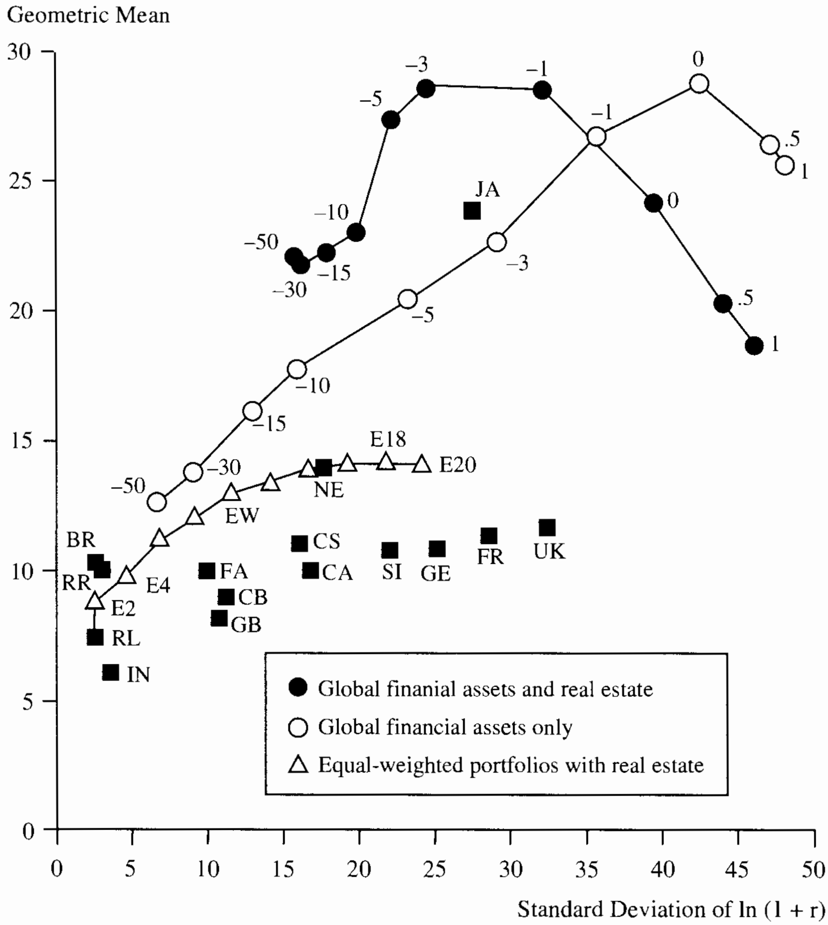
	Borrowing Precluded		Borrowing Permitted	
Portfolio	Geometric Mean	Standard Deviation*	Geometric Mean	Standard Deviation*
Common Stocks	10.6	16.0	10.6	16.0
Long-term Government Bonds	8.5	10.7	8.6	10.7
Long-term Corporate Bonds	9.2	11.1	9.2	11.1
Business Real Estate	10.2	3.0	10.2	3.0
Residential Real Estate	10.0	3.4	10.0	3.4
Farm Real Estate	10.0	10.0	10.0	10.0
French Equities	11.3	28.8	11.3	28.8
German Equities	10.7	25.3	10.7	25.3
Dutch Equities	13.6	17.9	13.6	17.9
Swiss Equities	10.8	22.2	10.8	22.2
British Equities	11.5	32.2	11.5	32.2
Japanese Equities	23.6	27.7	23.6	27.7
Canadian Equities	9.9	16.4	9.9	16.4
Lending	7.9	2.5	7.9	2.5
Inflation	6.3	3.2	6.3	3.2
Portfolio E2	9.0	2.9	9.0	2.9
Portfolio E4	10.1	4.9	10.1	4.9
Portfolio E6	11.1	7.1	11.1	7.1
Portfolio E8	12.0	9.5	12.0	9.5
Portfolio EW	12.9	11.8	12.9	11.8
Portfolio E12			13.2	14.4
Portfolio E14			13.5	16.9
Portfolio E16			13.7	19.4
Portfolio E18			13.9	21.8
Portfolio E20			13.9	24.2
U.S. Financial Assets				
Power −50	8.1	2.6	8.1	2.6
Power −30	8.2	2.7	8.2	2.7
Power −15	8.5	3.2	8.5	3.2
Power −10	8.7	3.8	8.7	3.8
Power −5	9.1	5.6	9.2	5.6
Power −3	9.5	7.5	9.7	7.8
Power −1	9.7	8.7	10.1	11.7
Power 0	9.3	9.2	9.9	14.7
Power 0.5	9.3	9.3	9.9	15.6
Power 1	9.7	9.6	−8.4	80.7

Table 8 ■ (continued)

Portfolio	Borrowing Precluded: Geometric Mean	Borrowing Precluded: Standard Deviation*	Borrowing Permitted: Geometric Mean	Borrowing Permitted: Standard Deviation*
U.S. Financial Assets Plus Real Estate				
Power −50	13.0	5.5	20.5	16.5
Power −30	13.1	6.3	20.8	18.4
Power −15	13.1	6.8	20.7	19.9
Power −10	12.9	7.1	21.1	21.5
Power −5	12.8	7.8	22.5	23.8
Power −3	13.1	8.1	23.5	24.8
Power −1	13.7	8.9	23.7	28.9
Power 0	13.6	9.1	17.7	43.5
Power 0.5	13.5	9.3	16.3	46.2
Power 1	13.9	8.8	−0.1	90.7
Global Financial Assets				
Power −50	12.2	6.8	12.2	6.8
Power −30	13.6	9.3	13.6	9.3
Power −15	15.6	13.3	15.8	13.4
Power −10	16.6	15.7	17.5	16.3
Power −5	17.7	19.0	20.5	23.6
Power −3	18.6	21.8	22.9	29.3
Power −1	20.8	26.3	26.6	35.6
Power 0	21.4	27.5	28.6	42.8
Power 0.5	21.6	27.6	26.8	47.8
Power 1	20.9	27.7	26.1	48.5
Global Financial Assets Plus Real Estate				
Power −50	13.7	5.4	24.8	16.0
Power −30	14.1	5.9	24.6	16.5
Power −15	15.0	7.6	24.9	18.0
Power −10	15.6	9.8	25.4	19.8
Power −5	17.4	15.3	27.7	22.4
Power −3	17.7	19.5	28.7	24.2
Power −1	19.2	25.2	28.2	32.4
Power 0	19.2	27.5	24.0	39.9
Power 0.5	20.1	27.9	20.1	44.4
Power 1	20.0	28.1	17.4	46.8

* Standard deviation of $\ln(1 + r)$

Figure 8 ■ Geometric means and standard deviations of annual returns. Global financial assets with and without real estate, 1970–1988. (Annual portfolio revision, borrowing permitted, 10-year estimating period.)



RL Riskfree Lending
GB Long-term U.S. Government Bonds
CB Long-term U.S. Corporate Bonds
CS U.S. Common Stocks (S & P 500)
FA Farm Real Estate
RR Residential Real Estate
BR Business Real Estate
CA Canadian Equities
FR French Equities
GE German Equities
JA Japanese Equities
NE Dutch Equities
SI Swiss Equities
UK British Equities
IN U.S. Inflation

EW Equal-weighted portfolio of risky assets in investment universe
E2 20% in EW, 80% in RL
E4 40% in EW, 60% in RL
E18 180% in EW, 80% in B
E20 200% in EW, 100% in B

Table 9 ■ Portfolio compositions and realized returns for power −3 with global ginancial assets plus real estate, 1970–1988. (Annual revision, borrowing permitted, 10-year estimating period.)

		Investment Fractions *												
Period	Return	RL	B	CS	GB	CB	BR	RR	FA	NE	SI	UK	JA	CA
1970	−10.1								0.1		0.2		0.5	
1971	31.6		−4.0						5.0					
1972	70.1		−4.0						5.0					
1973	111.3		−3.5						4.2				0.3	
1974	9.1		−1.1						1.6				0.5	1.0
1975	38.4		−2.1						2.7				0.4	
1976	61.2		−3.6						4.4				0.3	
1977	27.1		−3.7						4.6				0.2	
1978	61.5		−3.4						4.1				0.4	
1979	24.7		−2.3						2.9				0.4	
1980	14.2		−0.5						1.3				0.2	
1981	5.2								0.7				0.3	
1982	−17.1		−0.8						1.5				0.3	
1983	0.4								1.0					
1984	8.9							0.7					0.3	
1985	52.8		−2.2	0.2	0.1		2.0					0.4	0.6	
1986	75.3		−2.7		0.2		2.5			0.1		0.2	0.7	
1987	23.1		−2.9		0.3		2.7					0.3	0.5	
1988	26.9		−2.9		0.3		2.8						0.8	

RL Riskfree Lending
B Borrowing
CS U.S. Common Stocks (S & P 500)
GB Long-term U.S. Government Bonds
CB Long-term U.S. Corporate Bonds
BR Business Real Estate
RR Residential Real Estate
FA Farm Real Estate
NE Dutch Equities
SI Swiss Equities
UK British Equities
JA Japanese Equities
CA Canadian Equities
* French and German equities were not selected by this investor.

Transaction Costs

Compared to financial assets, transaction costs in buying and selling real estate are rather high, on the order of 5%–10% vs. 1% for financial assets. As noted, no attempt was made to formally take transaction costs into account in this exploratory study. One aspect which ameliorates their impact is the fact that holdings were subject to revision only once a year. It should also be noted, however, that the optimal strategy *ignoring* transactions costs led to either small or no portfolio adjustments most of the time, as Tables 3, 4, 6, 7 and 9 show—occasionally, though rather sizable asset shifts were undertaken. For example, the transaction costs incurred by the logarithmic investor in the U.S. only, no leverage case with real estate would have been small indeed (see Table 4).

Statistical Tests

There are a number of commonly accepted ways of testing for abnormal investment performance: (1) Jensen's (1968) test of selectivity, or microforecasting; (2) Henriksson and Merton's (1981) and Treynor and Mazuy's (1966) tests of market timing or macroforecasting; and (3) a paired t-test of the difference in investment returns. In view of the relatively long holding period (one year), the small number of observations, and the lack of a well-defined market portfolio when the universe extends beyond U.S. equities, only the paired t-test was employed in this study.

Turning to the paired t-test of the difference in investment returns, recall that terminal wealth w_0 in terms of beginning wealth w_n is given by

$$w_0 = w_n(1 + r_n)(1 + r_{n-1}) \ldots\ldots (1 + r_1)$$
$$= w_n \exp\left[\sum_{t=1}^{n} \ln(1 + r_t)\right].$$

Since returns compound multiplicatively, we employ the paired t-test for dependent observations to the annual (and additive) variables $\ln(1 + r_t)$. Thus, to compare the return series $r_1^1, \ldots, r_n^1$ with the return series $r_1^2, \ldots, r_n^2$ for two different strategies, we calculate the statistic

$$t = \frac{\bar{d}}{\sigma(d)/\sqrt{n}},$$

where

Table 10 ■ Paired t-tests of the differences in investment returns for the power policies.

	Borrowing Precluded			Borrowing Permitted		
	$\bar{d}$	$\sigma(d)$	t	$\bar{d}$	$\sigma(d)$	t
U.S. Financial Assets Plus Real Estate vs. U.S. Financial Assets, 1955–1988						
Power −50	0.034	0.052	3.9***	0.069	0.132	3.1***
Power −30	0.035	0.062	3.3***	0.071	0.135	3.1***
Power −15	0.032	0.071	2.6***	0.081	0.150	3.2***
Power −10	0.027	0.074	2.2**	0.088	0.168	3.0***
Power −5	0.021	0.081	1.5*	0.101	0.202	2.9***
Power −3	0.017	0.087	1.2	0.104	0.216	2.8***
Power −1	0.019	0.102	1.1	0.101	0.244	2.4**
Power 0	0.020	0.100	1.2	0.088	0.256	2.0**
Power 0.5	0.024	0.098	1.4*	0.075	0.316	1.4*
Power 1	0.025	0.097	1.5*	0.075	0.437	1.0
U.S. Financial Assets Plus Real Estate vs. U.S. Financial Assets, 1955–1967						
Power −50	0.028	0.039	2.7***	0.029	0.040	2.7***
Power −30	0.028	0.043	2.3**	0.037	0.049	2.7***
Power −15	0.017	0.049	1.3	0.050	0.077	2.4**
Power −10	0.011	0.045	0.9	0.057	0.101	2.0**
Power −5	0.002	0.037	0.2	0.062	0.117	1.9**
Power −3	−0.009	0.020	−1.6*	0.057	0.134	1.5*
Power −1	−0.010	0.027	−1.3	0.019	0.141	0.5
Power 0	−0.004	0.015	−1.0	−0.004	0.130	−0.1
Power 1	0.000	0.000	0.0	−0.020	0.110	−0.7
U.S. Financial Assets Plus Real Estate vs. U.S. Financial Assets, 1968–1988						
Power −50	0.038	0.060	2.9***	0.094	0.162	2.7***
Power −30	0.039	0.072	2.5**	0.093	0.166	2.6***
Power −15	0.041	0.082	2.3**	0.100	0.180	2.5***
Power −10	0.038	0.098	1.6*	0.125	0.240	2.4**
Power −3	0.034	0.107	1.4*	0.133	0.253	2.4**

Table 10 ■ (continued)

	Borrowing Precluded			Borrowing Permitted		
	$\bar{d}$	$\sigma(d)$	t	$\bar{d}$	$\sigma(d)$	t
U.S. Financial Assets Plus Real Estate vs. U.S. Financial Assets, 1968–1988						
Power −1	0.037	0.126	1.3*	0.152	0.281	2.5**
Power 0	0.037	0.124	1.4*	0.140	0.297	2.2**
Power 0.5	0.041	0.121	1.6*	0.123	0.385	1.5*
Power 1	0.041	0.121	1.6*	0.135	0.546	1.1
Global Financial Assets Plus Real Estate vs. Global Financial Assets, 1970–1988						
Power −50	0.013	0.085	0.7	0.106	0.167	2.8***
Power −30	0.005	0.092	0.2	0.093	0.167	2.4***
Power −15	−0.005	0.102	−0.2	0.075	0.188	1.8**
Power −10	−0.009	0.107	−0.4	0.065	0.222	1.3
Power −5	−0.003	0.092	−0.1	0.058	0.286	0.9
Power −3	−0.007	0.085	−0.4	0.046	0.307	0.7
Power −1	−0.014	0.072	−0.8	0.013	0.332	0.2
Power 0	−0.018	0.041	−1.9**	−0.037	0.285	−0.6
Power 0.5	−0.012	0.044	−1.2	−0.055	0.283	−0.9
Power 1	−0.008	0.034	−1.0	−0.071	0.234	−1.3

* (**, ***) — statistically significant at the .1 (.05, .01) level in a one-tailed test.

$$\bar{d} = \sum_{t=1}^{n} \frac{\ln(1 + r_t^1) - \ln(1 + r_t^2)}{n}$$

and $\sigma(d)$ is the standard deviation of $\ln(1 + r_t^1) - \ln(1 + r_t^2)$. In each case, the null hypothesis is that

$$E[\ln(1 + r_t^1)] = E[\ln(1 + r_t^2)]$$

while the alternative hypothesis is that

$$E[\ln(1 + r_t^1)] > E[\ln(1 + r_t^2)].$$

Table 10 shows the results of the paired t-test when the returns with and without real estate are compared for each risk attitude or power. It is evident that the inclusion of real estate in most cases improved returns at the 1% level of significance for the most risk-averse strategies, the only exception being the global case when leverage was unavailable. On the other hand, the presence of real estate had a mixed impact for the highly risk-tolerant strategies, and a more modest positive impact for the in-between risk attitudes. The most striking thing about these results is their consistency with the findings of previous studies: the most risk-averse strategies were also the greatest beneficiaries not only from domestic diversification among the major asset categories (Grauer and Hakansson 1982, 1985, 1986) but also from international diversification (Grauer and Hakansson 1987).

Results—Adjusted Version

The asset allocation strategies based on the six asset category universe (risk free lending, U.S. governments, U.S. corporates, U.S. equities, farm real estate and residential real estate, with borrowing permitted) were run over the full 1955–1988 period (corresponding to Table 5 and Figure 4) using the mean-variance approximation in equations (7) and (8) without adjustment and with two desmoothing adjustments. In the first adjustment, the estimated variance of farm real estate was increased by a factor of 2 and that of residential real estate by a factor of 9. In the second adjustment, both raw variances were multiplied by a factor of 9 (which is a rather drastic risk adjustment). The results are reported in Table 11 and in Figures 9 and 10.

Figure 9 shows the ex ante mean-variance efficient frontiers at the beginning of four different years (1965, 1972, 1973 and 1978).[17] Note that the desmoothed efficient frontier lies substantially below the unadjusted one in each case. (The nonlinearities of the frontiers are due to the constraints on borrowing, short sales, and the differences between borrowing and lending rates.)

Comparing the first two columns in the bottom panel of Table 11 with the first two columns in Table 5 (in the ex post case), it is evident that the mean-variance approximation for the higher powers is better than for the lower ones, which is consistent with the results of Grauer and Hakansson (1993a) under annual revision. For power 0.5, for example, the geometric means and standard deviations for the approximation were (18.8, 27.5) vs. (17.8, 28.9) for the exact formulation. For power −50, however, the corresponding numbers are (11.3, 7.7) vs. (14.2, 12.6), a much poorer result.

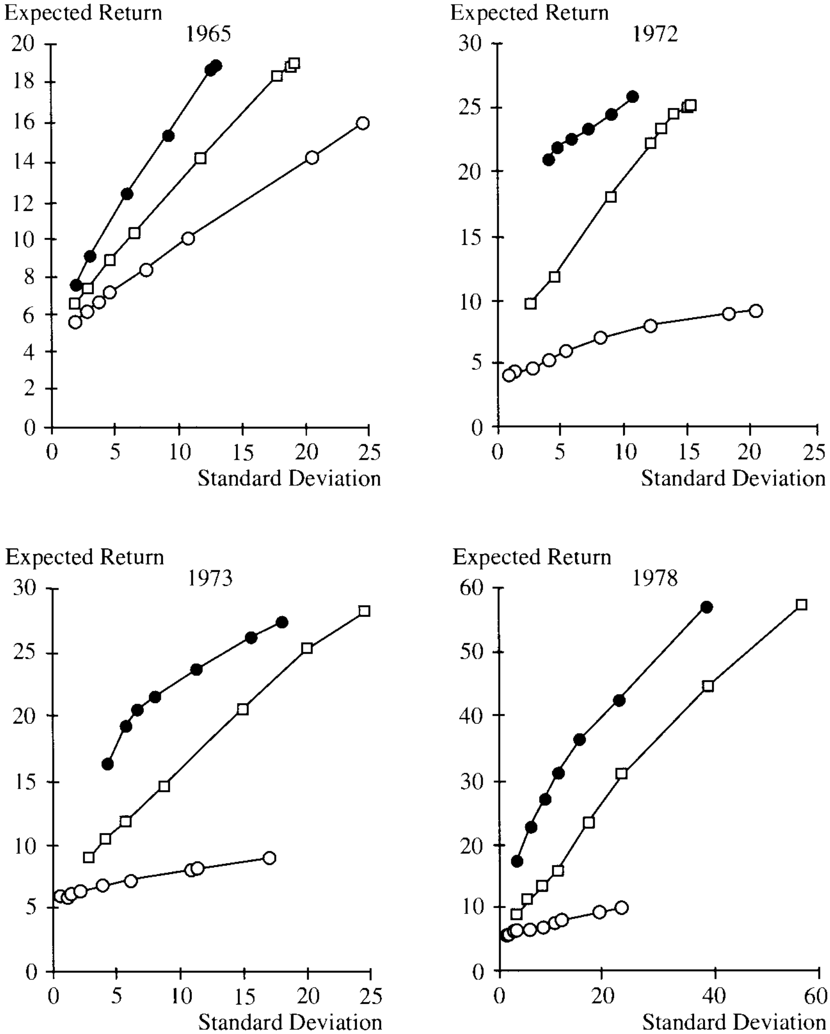
[17] The (large) differences between the three frontiers are consistent with the differences in realized returns for the three cases.

Table 11 ■ Geometric means and standard deviations of annual returns of U.S. financial assets plus real estate based on the mean-variance approximation to the power policies when the variances of residential and farm real estate are adjusted, 1955–1988. (Annual portfolio revision, borrowing permitted, 8-year estimating period.)

	Unadjusted Case		First Adjustment[a]		Second Adjustment[b]	
	Geometric Mean	Standard Deviation*	Geometric Mean	Standard Deviation*	Geometric Mean	Standard Deviation*
Common Stocks	10.4	15.3	10.4	15.3	10.4	15.3
Long-term Government Bonds	4.9	9.6	4.8	9.6	4.9	9.6
Long-term Corporate Bonds	5.5	9.7	5.5	9.7	5.5	9.7
Residential Real Estate	8.1	3.4	8.1	3.4	8.1	3.4
Farm Real Estate	9.4	7.6	9.4	7.6	9.4	7.6
Lending	6.1	2.9	6.1	2.9	6.1	2.9
Inflation	4.5	3.2	4.5	3.2	4.5	3.2
Portfolio E2	6.5	2.8	6.5	2.8	6.5	2.8
Portfolio E4	6.9	3.0	6.9	3.0	6.9	3.0
Portfolio E6	7.3	3.5	7.3	3.5	7.3	3.5
Portfolio E8	7.7	4.2	7.7	4.2	7.7	4.2
Portfolio EW	8.0	5.0	8.4	5.0	8.4	5.0
Portfolio E12	8.0	6.0	8.0	6.0	8.0	6.0
Portfolio E14	7.9	6.9	7.9	6.9	7.9	6.9
Portfolio E16	7.8	7.7	7.8	7.7	7.8	7.7
Portfolio E18	7.8	8.6	7.8	8.6	7.8	8.6
Portfolio E20	7.6	9.5	7.6	9.5	7.6	9.5
Power −50	11.3	7.7	8.4	3.4	7.1	2.6
Power −30	12.6	9.1	9.4	4.6	7.7	2.9
Power −15	14.1	10.6	11.4	7.2	8.7	3.9
Power −10	15.5	12.0	12.8	9.8	9.6	5.1
Power −5	18.3	15.3	15.5	13.6	11.2	8.4
Power −3	20.0	17.9	17.9	16.4	12.4	11.5
Power −1	21.4	20.6	20.6	19.8	15.6	15.2
Power 0	21.2	22.9	21.5	21.5	18.1	18.5
Power 0.5	18.8	27.5	21.4	24.4	19.7	22.7
Power 1	−100.0		−100.0		−100.0	

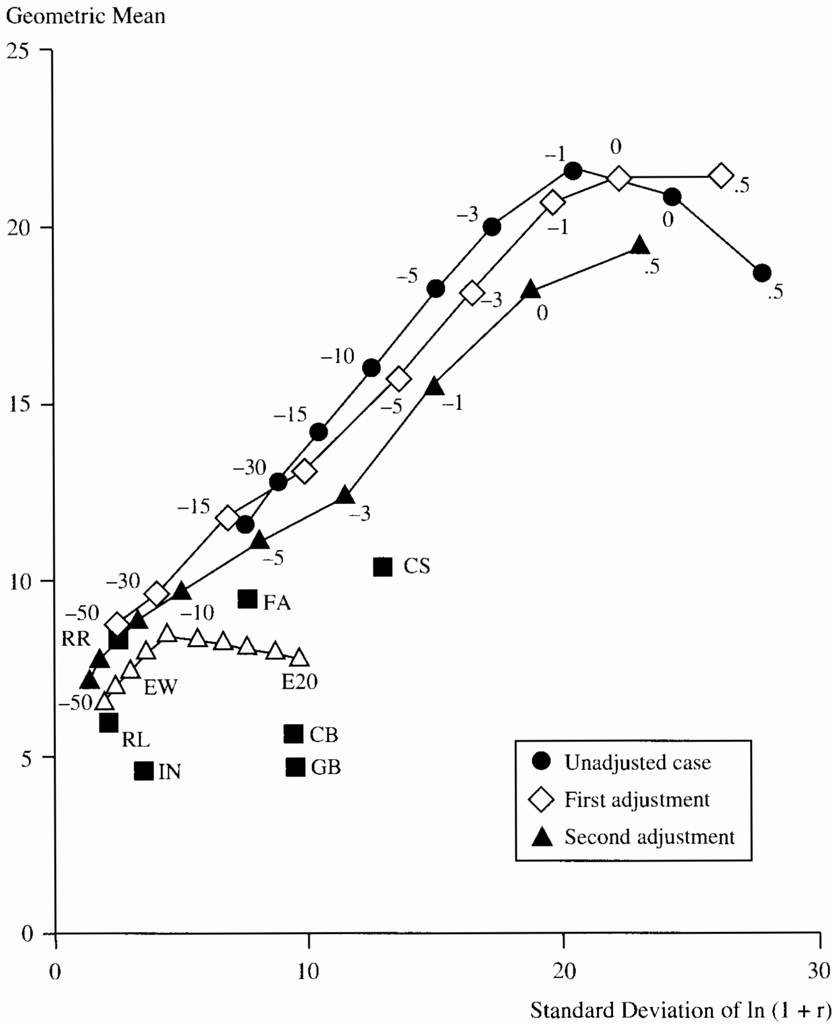
[a] The variances of residential and farm real estate were multiplied by 9 and 2 respectively.
[b] The variances of residential and farm real estate were multiplied by 9 and 9 respectively.
* Standard deviation of $\ln(1 + r)$.

Figure 9 ■ Ex-ante mean-variance efficient-frontiers for selected years. U.S. Financial assets with and without real estate. (Borrowing permitted, 8-year estimating period.)



Key to symbols: ○ Financial assets only;
● Financial assets and real estate unadjusted;
□ Financial assets and real estate adjusted (first adjustment).

Figure 10 ■ Geometric means and standard deviations of annual returns of U.S. financial assets plus real estate based on the mean-variance approximation to the power policies when the variances of residential and farm real estate are adjusted, 1955–1988. (Annual portfolio revision, borrowing permitted, 8-year estimating period.)



RL	Riskfree Lending		
GB	Long-term U.S. Government Bonds	EW	Equal-weighted portfolio of risky assets in investment universe
CB	Long-term U.S. Corporate Bonds		
CS	U.S. Common Stocks (S & P 500)	E20	200% in EW, 100% in B
FA	Farm Real Estate		
RR	Residential Real Estate		
IN	U.S. Inflation		

As Figure 10 and the bottom panel of Table 11 show, the impacts of the two risk adjustments were greatest for the more risk-averse strategies. Both the geometric mean and the volatility of returns were greatly reduced. The effect on the more risk tolerant strategies, however, was surprisingly small. For the growth-optimal (power 0) strategy, the impact of the first adjustment is negligible, as is the effect of the second adjustment on power 0.5.

The above results in returns space were born out in policy space as well. The substantially reduced returns and risk of the very risk-averse strategies attained under the two risk adjustments were brought about by sizable shifts of investment flows from real estate to Treasury Bills. For the more risk tolerant investors, however, shifts away from real estate were small even under the second adjustment.

Under valid probability estimates, the growth-optimal (power 0) strategy asymptotically attains the highest geometric mean return among all possible strategies (that do not converge to the power 0 strategy; for example, Thorp 1975). On this basis, Table 11 and Figure 10 suggest that the first adjustment, which increases the standard deviation of farm real estate returns by a factor of $\sqrt{2}$ (to about 10.7%) and that of residential real estate by a factor of 3 (to about 10.3%), may be the most meritorious of the three cases examined. These numbers jibe well with the results of Ross and Zisler (1991) and Geltner (1991). Nevertheless, much work remains to be done in the area of desmoothing of real estate returns series.

Conclusion

This paper compares the investment policies and returns for portfolios of stocks and bonds with and without real estate. The portfolios were generated via the discrete-time dynamic investment model on the basis of the empirical probability assessment approach applied to past realized returns. Both a domestic and a global setting were examined, with and without possibility of leverage and with and without desmoothing of real estate returns. With respect to real estate data risk adjustment, a much longer period than previously addressed was studied, with interesting findings.

Our principal findings are: (1) the gains from adding real estate on a semi-passive (equal-weighted) basis to portfolios of either U.S. or global financial assets were relatively modest; in contrast, (2) the gains from adding real estate to the universe of U.S. financial assets under an active strategy were remarkably large (in some cases highly statistically significant), especially for the very risk-averse strategies; (3) the gains from adding (U.S.) real estate to a universe of global financial assets under an active strategy were mixed, although generally favorable for the highly risk-averse strategies; (4) correcting for second-moment smoothing had a relatively small impact for the more risk-tolerant strategies; and (5) there was some evidence that desmoothing resulted in improved probability estimates. In the area of real estate risk adjustment, it is evident that much exploration remains to be done.

It is perhaps noteworthy that even though probability estimates in the model used in the study were based on the (joint) realizations of returns over the *previous* eight or ten years, financial assets were favored in the fifties, sixties and eighties (see Tables 3, 4, 6, 7 and 9) while real estate was the principal investment outlet in the seventies—a period of high inflation and in which the U.S. stock market experienced virtually no price appreciation.

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