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On Timing The Market: the Empirical Probability Assessment Approach with an Inflation Adapter

Robert R. Grauer and Nils H. Hakansson

Abstract

Recent studies have documented varying degrees of predictability and mean reversion in stock returns but the question of how they might be exploited remains relatively open. This article applies dynamic portfolio theory to the construction and rebalancing of portfolios principally composed of stocks and cash or borrowing. Probability assessments were based on (all moments of) recent past returns, both in raw form and with an inflation adapter. The inflation adapter had little impact prior to the mid-sixties but markedly improved realized portfolio returns over the 1966–88 subperiod. Some excess returns exceeded one percent per quarter but the various performance tests applied reached contradictory conclusions concerning over which period the null hypothesis should be rejected.

1 Introduction

Recent studies have documented varying degrees of predictability of stock returns as well as mean-reversion of returns over intermediate to long holding periods (see e.g. Keim and Stambaugh (1986), French, Schwert, and Stambaugh (1987), Fama and French (1988a,b), and Poterba and Summers (1988))[1]. This raises the question of whether and how such patterns in returns might be exploited by a price-taking investor. The purpose of this article is to examine the extent to which (all moments of) recent past returns are useful (or not useful) in revising ones's portfolio while moving forward in time on the basis of the discrete-time dynamic investment model[2].

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[1] Some schools of practitioners have long subscribed to the view that stock prices are at least partially predictable from past prices and related summary statistics via what is commonly called 'technical analysis'. While generally pooh-poohed by academics (see e.g. Malkiel (1990)), a recent study employing two simple and popular technical trading rules provides formal evidence in support of their value – see Brock, Lakonishok, and LeBaron (1992).

[2] For a review of alternative approaches, including stochastic programming, to the dynamic investment problem, see Mulvey and Ziemba (1995).

In earlier papers, Grauer and Hakansson (1982, 1984, 1986) applied the dynamic portfolio theory of Mossin (1968), Hakansson (1971, 1974), Leland (1972), Ross (1974), and Huberman and Ross (1983) in conjunction with the empirical probability assessment approach (EPAA) to construct and rebalance portfolios composed of US stocks, corporate bonds, government bonds, and a risk-free asset. Borrowing was ruled out in the first article, while margin purchases were permitted in the other two. The probability distributions used were naively estimated from *past* realized returns in the Ibbotson and Sinquefield and Ibbotson data bases, and both annual and quarterly holding periods were employed from the mid-thirties forward. The results of all three papers revealed that the gains from active diversification among the major asset categories were substantial, especially for the highly risk-averse strategies. (Two other papers, Grauer and Hakansson (1987) and Grauer and Hakansson (1995b), document even larger gains from international diversification and from the inclusion of real estate into the investment universe; see also Jorion (1989).) In addition, they found evidence of substantial use of, and gains from, margin purchases for the more risk-tolerant strategies from the mid-thirties to the mid-sixties. The third paper also showed that small stocks, while sometimes totally ignored, entered even the most risk-averse portfolios most of the time.

The empirical probability assessment approach may be modified by correcting for estimation error either in the means or in the variances (or both). In Grauer and Hakansson (1995a), a James–Stein, a Bayes–Stein, and a third estimator for adjusting the means were employed with mixed results compared to the no adjustment case. In Grauer and Hakansson (1995b), the desmoothing of the real estate input series resulted in a modest improvement.

In the present article, the raw joint empirical distribution approach to generating probability assessments is refined by the inclusion of an inflation adapter. The inflation adapter at any point in time is based on a simple regression of past returns on inflation. Specifically, the difference between the observed risk-free lending rate in the coming period and its average over the estimating period multiplied by the estimated inflation sensitivity coefficient is added to the raw probability distribution for each asset in the period. The effect is to change the projected mean returns, leaving all other moments unchanged.

Use of the inflation adapter substantially changed the portfolios selected, especially for the more risk-tolerant strategies during the highly inflationary 1966–82 sub-period. Superficially, the strategies give the appearance of intensified 'market timing' activities. In return space, the inflation adapter also led to uniformly higher geometric means and lower standard deviations of realized returns.

To minimize the noise factor in the performance measurement process, the model's portfolio selection possibilities is reduced to the two asset case (stocks and cash) with quarterly revision, for the most part. Stocks are represented by the Center for Research in Security Prices (CRSP) value-weighted index or market portfolio. Cash means either a long position in 90-day Treasury bills or borrowing at the call money rate +1%. A comparison with the performance of 130 mutual funds, and with the performance of active strategies generated from an expanded investment universe composed of long-term US government and corporate bonds, the S&P 500 index, and an index of small stocks, is also made for the 1968–82 period.

In all, seven tests of performance were employed: the Jensen test, the Henriksson–Merton (HM) test, the Treynor–Mazuy (TM) test, the paired *t*-test, and variants thereof. Overall, there is considerable evidence of significant positive abnormal returns or market timing ability with, and to some extent without, the inflation adapter. This is somewhat remarkable since only *past* realized returns were used as inputs. For example, based on the Jensen test, each of the risk tolerances that employed the inflation adapter yielded significant abnormal returns over the full 1934–88 period, as well as over the 1966–88 sub-period when the fourth quarter of 1987 was excluded. The market timing tests indicated that there was highly significant positive market timing ability in any sub-period beginning in 1966 that excluded the fourth quarter of 1987. Similarly, the paired *t*-tests showed that a number of the active strategies earned significantly higher returns than selected benchmarks over the 1934–88 period, as well as over the 1966–88 sub-period when the fourth quarter of 1987 was excluded from the sample.

2 Theory

Despite explosive development over three decades, and extensive application to the construction of equity portfolios, modern portfolio theory has found only modest use in the larger portfolio context, often referred to as the asset allocation problem – the choice of the proportions to be held in the major categories of common stocks and in different types of bonds, money market instruments, real estate, and foreign securities. There are several reasons for this. First, extant portfolio theory, being principally based on the mean-variance model, is single-period in nature, whereas the asset allocation problem accents the multiperiod, sequential nature of investment decisions[3]. On top of this, since the universe of interest extends well beyond common stocks, extant betas are too narrowly defined to be useful, and the appropriate betas are not easily estimated because of data problems concerning the market weights of bonds, for example. At the other extreme, continuous-time

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[3] Tests showing the superiority of dynamic models appear in Carino, Kent, Myers, Stacy, Sylvanus, Turner, Watanabe, and Ziemba (1994).

portfolio theory is somewhat intractable in a world of non-trivial transactions costs, although discrete-time approximation applications have been made (see e.g. Brennan and Schwartz, this volume). Finally, many extant models rely heavily on narrow classes of theoretical (stationary) return distributions, with limited ability to capture the richness of joint, real-world stochastic processes.

There is, however, a middle category of investment models, usually classified under the heading of discrete-time dynamic portfolio theory, which has been largely ignored in portfolio selection applications. This is so despite the fact that these models have a strong foundation in theory and lend themselves naturally to the problem of rebalancing portfolios over many periods (up to 220 quarters in the present study). An additional virtue of these models is that they can handle general nonstationary return distributions.

To review, consider the simplest reinvestment problem, in which the market is perfect and returns are independent over time but otherwise arbitrary and not necessarily stationary. The investor has a preference function U_0 (with $U_0' > 0$, $U_0'' < 0$) defined on wealth w_0 at some terminal point (time 0). Let w_n denote the investor's wealth with n periods to go, r_{in} the return on asset i in period n, z_{in} the amount invested in asset i in period n (with $i = 1$ being the safe asset), and $U_n(W_n)$ the relevant (unknown) utility of wealth with n periods to go. At the end of period n (time $n-1$), the investor's wealth is

$$w_{n-1}(z_n) = \sum_{i=2}^{M}(r_{in} - r_{1n})z_{in} + w_n(1 + r_{1n}),$$

where $z_n = (z_{1n}, \ldots, z_{Mn})$ and M is the number of securities.

Consider the portfolio problem with one period to go. The investor, with w_1 to invest, must solve

$$\max_{z_1|w_1} \mathrm{E}\left[U_0\left(w_0\left(z_1\right)\right)\right] \equiv U_1\left(w_1\right).$$

Clearly, $U_1(w_1)$ represents the highest attainable expected utility level from capital level w_1 at time 1, and thus the 'derived' utility of w_1. Employing the induced utility function $U_1(w_1)$, the portfolio problem with two periods to go becomes

$$U_2(w_2) \equiv \max_{z_2|w_2} \mathrm{E}\left[U_1\left(w_1(z_2)\right)\right].$$

Thus, with n periods to go, we obtain (the recursive equation)

$$U_n(w_n) = \max_{z_n|w_n} \mathrm{E}\left[U_{n-1}\left(w_{n-1}(z_n)\right)\right], \quad n = 1, 2, \ldots .$$

Examining the above system, it is evident that the induced utility of current wealth, $U_n(w_n)$, generally depends on 'everything', namely the terminal utility function U_0, the joint distribution functions of future returns, and future

interest rates. There is, however, a special case in which $U_n(w_n)$ depends only on U_0. This occurs [Mossin (1968)] if and only if $U_0(w_0)$ is isoelastic, i.e., if and only if

$$U_0(w_0) = \frac{1}{\gamma}w^{\gamma}, \quad \gamma < 1.$$

(Note that for $\gamma = 0$, $U_0(w_0) = \ln w_0$.) $U_n(w_n)$ is now a positive linear transformation of $U_0(w_n)$, i.e., we can write

$$U_n(w_n) = \frac{1}{\gamma}w_n^{\gamma}. \tag{1}$$

For these preferences, the optimal investment policy $z_{n\gamma}^*(w_n)$ is proportional to wealth, i.e.,

$$z_{in\gamma}^*(w_n) = x_{in\gamma}^* w_n, \quad \text{all } i, \tag{2}$$

where the $x_{in\gamma}^*$ are constants. It is also completely *myopic* since it only depends on U_0 and the current period's return structure and *not* on returns beyond the current period. Both of these properties hold only for the family 1, which is also the only class of preferences exhibiting constant relative risk aversion[4]. Finally, 2 also implies that the utility of wealth relatives, $V_n(1+r_n)$, is of the same form only for this family, i.e.,

$$U_n(w_n) = \frac{1}{\gamma}w^{\gamma} \Longleftrightarrow V_n(1 + r_n) = \frac{1}{\gamma}(1 + r_n)^{\gamma}.$$

While the above properties are interesting, they are clearly rather special. However, the isoelastic family's influence extends far beyond its numbers. As shown by Hakansson (1974) (see also Leland (1972), Ross (1974), and Huberman and Ross (1983)), there is a very broad class of terminal utility functions $U_0(w_0)$ for which the induced utility functions U_n converge to an isoelastic function, i.e., for which

$$U_n(w_n) \longrightarrow \frac{1}{\gamma}w^{\gamma}, \quad \text{for some } \gamma < 1. \tag{3}$$

Hakansson (1974) has also shown that 3 is usually accompanied by convergence in policy, i.e.,

$$z_n^* \longrightarrow x_{n\gamma}^* w_n.$$

Thus, the objectives given by 1 are quite robust and encompass a broad variety of different goal formulations for investors with intermediate to long-term investment horizons[5]. In particular, class 1 spans a continuum of risk attitudes all the way from risk neutrality ($\gamma = 1$) to infinite risk aversion ($\gamma = -\infty$).[6]

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[4] This measure is defined as $-wU_n''(w)/U_n'(w)$ and equals $1 - \gamma$ for the class 1.

[5] The simple reinvestment formulation does ignore consumption of course.

[6] A plot of the functions $\frac{1}{\gamma}(1 + r)^{\gamma}$ for several values of γ was given in Grauer and Hakansson (1982, p. 42).

Having selected our model, we turn next to what we need to operate it. The major input to the model is an estimate of next period's *joint* return distribution for the risky asset categories[7]. In several previous studies, we based this estimate on the empirical probability assessment approach (EPAA). In this approach, the realized returns of the most recent n periods are recorded; each of the n *joint* realizations is then assumed to have probability $1/n$ of occurring in the coming period. Thus, estimates were obtained on a moving basis and used in raw form without adjustment of any kind. On the other hand, since the whole joint distribution was specified and used, there was no information loss; all moments and correlations were taken into account. The empirical distribution of the past n periods is optimal if the investor has no information about the form and parameters of the true distribution, but believes that this distribution went into effect n periods ago, see Bawa, Brown and Klein (1979, p. 160).

3 The Inflation Adjustment

It has been well documented that asset returns are sensitive to the rate of inflation, e.g. Coleman (1966), Fama (1975), Fama and Schwert (1977). In particular, there is strong evidence that US Treasury bills are a perfect hedge against anticipated inflation and that returns on long-term bonds and common stocks are negatively related to at least unanticipated inflation.

In this article, the empirical probability assessment approach is modified by the addition of an inflation adapter. This adapter is designed to adjust the raw distribution for anticipated inflation, as reflected in the three-month Treasury bill rate, at the time of investment, t.

Specifically, let $r_{i\tau}$ be the realized return on asset category i in period τ, I_{τ^-} the realized inflation rate in the (three-month) period ending one month prior to the end of period (calendar quarter) τ (since the inflation rate is published with a lag), and r_{Lt} the risk-free lending rate in period t, for which the decision is to be made. The regression

$$r_{i\tau} = a_i^t + b_i^t I_{\tau^-} + e_{i\tau}$$

is run for each i over the estimating period $t-n$ to $t-1$ to obtain the 'rolling' estimated coefficients $\hat{a}_i^t$ and $\hat{b}_i^t$. We take as our base for estimating inflation's impact over the next period the difference between the observed risk-free lending rate, r_{Lt}, and its average, $\bar{r}_{Lt}$, over the estimating period, i.e., $r_{Lt} - \bar{r}_{Lt}$. The quantity

$$\hat{b}_i^t(r_{Lt} - \bar{r}_{Lt})$$

is then added to the raw probability distribution for each asset i in period t. The effect is to change the anticipated mean returns, leaving all other moments unchanged.

4 Calculations

The model used can be summarized as follows. At the beginning of each period t, the investor chooses a portfolio, x_t, on the basis of some member, γ, of the family of utility functions for returns r given by

$$V(1+r) = \frac{1}{\gamma}(1+r)^\gamma. \tag{4}$$

This is equivalent to solving the following nonlinear programming problem in each period t:

$$\max_{x_t} \mathrm{E}\left[\frac{1}{\gamma}(1+r_t(x_t))^\gamma\right] = \max_{x_t} \sum_s \pi_{ts}\frac{1}{\gamma}(1+r_{ts}(x_t))^\gamma \tag{5}$$

subject to

$$x_{it} \geq 0,\ x_{Lt} \geq 0,\ x_{Bt} \leq 0, \qquad \text{all } i, \tag{6}$$

$$\sum_i x_{it} + x_{Lt} + x_{Bt} = 1, \tag{7}$$

$$\sum_i m_{it}x_{it} \leq 1, \tag{8}$$

$$\Pr(1 + r_t(x_t) \geq 0) = 1, \tag{9}$$

where

$r_{ts}(x_t) = \sum_i x_{it}r_{its} + x_{Lt}r_{Lt} + x_{Bt}r_{Bt}^d$, is the (*ex ante*) return on the portfolio in period t if state s occurs,

$\gamma \leq 1$ = a parameter that remains fixed over time,

x_{it} = the amount invested in risky asset category i in period t as a fraction of own capital,

$x_t = (x_{1t}, \ldots, x_{nt}, x_{Lt}, x_{Bt})$,

r_{it} = the anticipated total return (dividend yield plus capital gains or losses) on asset category i in period t,

r_{Lt} = the return on the risk-free asset in period t,

r_{Bt}^d = the borrowing rate at the time of the decision at the beginning of period t,

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[7] For a comprehensive overview of the issues and problems associated with the estimation of return distributions see Bawa, Brown and Klein (1979).

m_{it} = the initial margin requirement for asset category i in period t expressed as a fraction, and

π_{ts}= the probability of state s at the end of period t, in which case the random return r_{it} will assume the value r_{its}.

Constraint 6 rules out short sales and 7 is the budget constraint. Constraint 8 serves to limit borrowing (when available) to the maximum permissible under the margin requirements that apply to the various asset categories. Finally, constraint 9 rules out any (*ex ante*) probability of bankruptcy. On the basis of the probability estimation method described earlier, the (sequential) solution of the portfolio problem may be described as follows. Suppose quarterly revision is used. Then, at the beginning of quarter t, the portfolio problem 5–9 for that quarter uses the following inputs – the (observable) risk-free return for quarter t, the (observable) call money rate +1% at the beginning of quarter t, and the (observable) realized returns for each of the risky assets for the previous n quarters, and the realized inflation rates for the previous n quarters. Each joint realization (whether inflation adjusted or not) in quarters $t-n$ through $t-1$ is given probability $1/n$ of occurring in quarter t.

With these inputs, the portfolio weights for the various asset categories and the proportion of assets borrowed are calculated by solving the nonlinear programming system 5–9; the algorithm employed is described in Best (1975). At the end of quarter t, the realized returns on each of the risky assets are observed, along with the *realized* borrowing rate r_{Bt} (which may differ from the decision borrowing rate r^d_{Bt})[8]. Then, using the weights selected at the beginning of the quarter, the realized return on the portfolio chosen for quarter t is recorded. The cycle is then repeated in all subsequent quarters[9].

All reported returns are gross of transaction costs and taxes and assume that the investor in question had no influence on prices. There are several reasons for this approach. First, we wish to follow precedent and keep the complications to a minimum. Second, the return series used as inputs and for comparisons also excludes transaction costs (for reinvestment of dividends) and taxes. Third, many investors are tax-exempt and various techniques are available for keeping transaction costs low. Finally, since the proper treatment of these items is nontrivial, they are better left to a later study.

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[8] The realized borrowing rate r_{Bt} was calculated as the average of the monthly realized rates.

[9] If $n = 32$ under quarterly revision, then the first quarter for which a portfolio can be selected is $b + 32$, where b is the first quarter for which data is available.

5 Data

The risk-free asset was assumed to be 90-day US Treasury bills maturing at the end of the quarter; we used the *Survey of Current Business* and *The Wall Street Journal* as sources. The total returns on the value-weighted market portfolio were obtained from the monthly Center for Research in Securities Prices (CRSP) data file. Margin requirements for stocks were obtained from the *Federal Reserve Bulletin*[10].

As noted, the borrowing rate was assumed to be the call money rate +1%; for *decision* purposes (but not for rate of return calculations), the applicable beginning of period rate, r^d_{Bt}, was viewed as persisting throughout the period and thus as risk-free. For 1934–76, the call money rates were obtained from the *Survey of Current Business*; for later periods, *The Wall Street Journal* was the source.

As a benchmark against which to judge the performance of the active policies, we examined the performance of 130 open-ended mutual funds over a common time period using quarterly data from January 1968 to December 1982. The returns data include all dividends paid by the funds and are net of all management costs and fees. The data set, provided by David Modest and Bruce Lehmann, updates Henriksson's (1984) data base by two years and one month and includes an additional fourteen funds. (See Lehmann and Modest (1987) and Henriksson (1984) for more detailed descriptions of the data base.) For comparative purposes, we also examined the performance of the active strategies generated from an investment universe that included long-term US government bonds, long-term US corporate bonds, the S&P 500 index, and an index of small stocks. The source of this data set was *Stocks, Bonds, Bills, and Inflation 1989 Yearbook* published by Ibbotson Associates, Inc.

6 Results

Because of space limitations, only a portion of the results can be reported here. However, Tables 1 through 8 and Figures 1 through 4 provide a fairly representative sample of our findings.

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[10] There was no practical way to take maintenance margins into account in our programs. In any case, it is evident from the results that they would come into play only for the more risk-tolerant strategies, even for them only occasionally, and that the net effect would be relatively neutral.

6.1 The Portfolio Returns

Table 1 shows the geometric means and standard deviations[11] of the realized annual returns for 16 strategies corresponding to γ's in 1 ranging from −75 (extremely risk averse) to 1 (risk neutral), with and without the inflation adapter, for the 55-year period 1934–88. The estimating period was 32 quarters. Recall that only two assets could be chosen in each period, the CRSP value-weighted index and a risk-free asset (Treasury bills). Panel A shows the results when no borrowing was permitted, while Panel B reports the returns when margin purchases were allowed. Finally, Panel C shows the return characteristics of various benchmarks: risk-free lending (RL), the CRSP value-weighted index (VW), inflation, and a set of fixed-weight (rebalancing) portfolios. Thus, V4 represents a portfolio which is always rebalanced to 40% in the index (VW) and 60% in risk-free lending RL at the beginning of each period. Similarly, V18 always invests 180% of its capital in the index by borrowing 80%, unless margin requirements put a lower cap on borrowed funds.

Figure 1 plots the geometric means and standard deviations of the realized annual returns for risk-free lending, inflation, and the value-weighted CRSP index (see squares), for the up and down-levered value-weighted CRSP index (see triangles), and, *for the borrowing case only*, for the 16 powers with the inflation adapter (see black dots), and for the 16 active strategies without the inflation adapter (see diamonds).

As Figure 1 shows (for the period 1934–88 when the compound inflation rate was 4.10 percent per annum), the benchmarks marginally outperformed the more risk-averse active strategies, while the less risk-averse active strategies clearly did better than the fixed-weight strategies. Moreover, Table 1 shows that in the inflation adapter case, with borrowing precluded, the non-negative powers attained higher geometric mean returns than the market with less standard deviation. Furthermore, the returns with and without the inflation adapter were quite similar. However, with the exception of the −2 and −3 power strategies with borrowing permitted, the returns generated by the inflation adjusted strategies strictly 'dominated' the unadjusted strategies in the sense that each inflation adjusted strategy (with or without borrowing) earned a higher geometric mean return with equal to or less standard deviation than the corresponding unadjusted strategy.

Table 2 and Figure 2 show the results for the 1966–88 sub-period, which was characterized by a compound inflation rate of 5.96 percent per annum. In this period five observations stand out. First, the active strategies clearly dominated the benchmarks. Second, the differences between using and not

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[11] The table reports the standard deviation of the log of unity plus the rate of return. This quantity is very similar to the standard deviation of the rate of return for return levels less than 25%.

Table 1. Geometric Means and Standard Deviations of Annual Returns for sixteen Power Policies with and without the Inflation Adapter, 1934–1988 Treasury Bills and CRSP Value-Weighted Index (Quarterly portfolio revision, 32–quarter estimating period)

With Inflation Adapter			Without Inflation Adapter		
Portfolio	Geom. Mean	Std. Dev.	Portfolio	Geom. Mean	Std. Dev.
Panel A: Borrowing Precluded					
Power −75	4.33	3.45	Power −75	4.32	3.45
Power −50	4.56	3.60	Power −50	4.54	3.62
Power −30	5.01	4.13	Power −30	4.97	4.19
Power −20	5.53	5.06	Power −20	5.48	5.17
Power −15	6.04	6.08	Power −15	6.00	6.18
Power −10	6.89	7.69	Power −10	6.82	7.97
Power −7	7.54	8.94	Power −7	7.48	9.12
Power −5	8.31	9.78	Power −5	8.22	9.86
Power −3	9.32	10.44	Power −3	9.19	10.66
Power −2	10.03	10.67	Power −2	9.95	11.07
Power −1	10.66	11.12	Power −1	10.54	11.66
Power 0	11.40	11.98	Power 0	11.05	12.58
Power .25	11.52	12.23	Power .25	11.13	12.90
Power .5	11.79	12.90	Power .5	11.27	13.72
Power .75	11.72	14.22	Power .75	10.92	14.95
Power 1	12.12	14.36	Power 1	11.22	15.10
Panel B: Borrowing Permitted					
Power −5	8.81	11.57	Power −5	8.61	11.93
Power −3	10.02	14.16	Power −3	10.09	14.29
Power −2	10.85	15.29	Power −2	11.04	15.31
Power −1	12.26	16.83	Power −1	12.04	17.12
Power 0	14.27	18.35	Power 0	13.78	18.78
Power .25	14.74	19.38	Power .25	14.19	19.78
Power .5	15.11	21.32	Power .5	14.57	22.01
Power .75	15.73	23.85	Power .75	15.21	24.79
Power 1	15.82	27.58	Power 1	15.57	28.05
Panel C: Benchmarks					
RL	3.86	3.44	V12	11.74	20.45
V2	5.54	4.18	V14	12.18	24.11
V4	7.12	6.88	V16	12.46	27.87
V6	8.58	10.08	V18	12.58	31.74
V8	9.93	13.44	V20	12.51	35.76
VW	11.16	16.89	Inflation	4.10	3.73

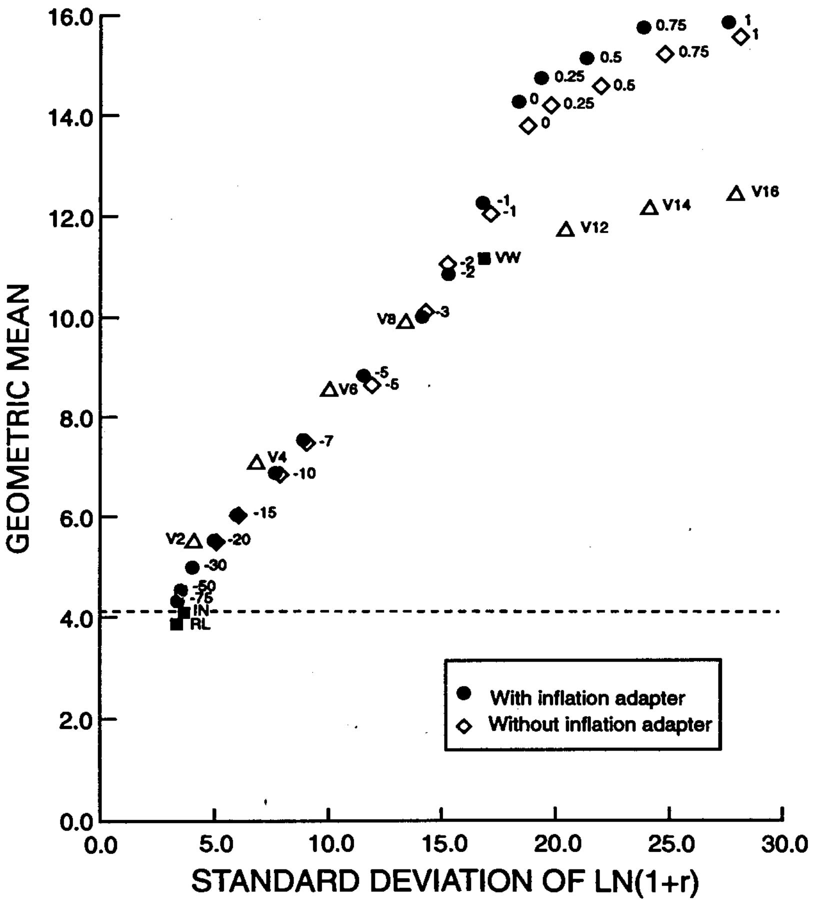

Figure 1. Geometric Means and Standard Deviations of Annual Returns for the Power Policies with and without the Inflation Adapter, Borrowing Permitted, 1934–1988 Treasury Bills and CRSP Value-Weighted Index (Quarterly portfolio revision, 32–quarter estimating period)

using the inflation adapter were somewhat larger. Third, with the exception of the −2 power with borrowing permitted, the inflation-adjusted strategies strictly 'dominated' the unadjusted strategies. Fourth, all the inflation adjusted active strategies less risk averse than the −7 power earned higher geometric mean rates of return, coupled with less variability, than the CRSP

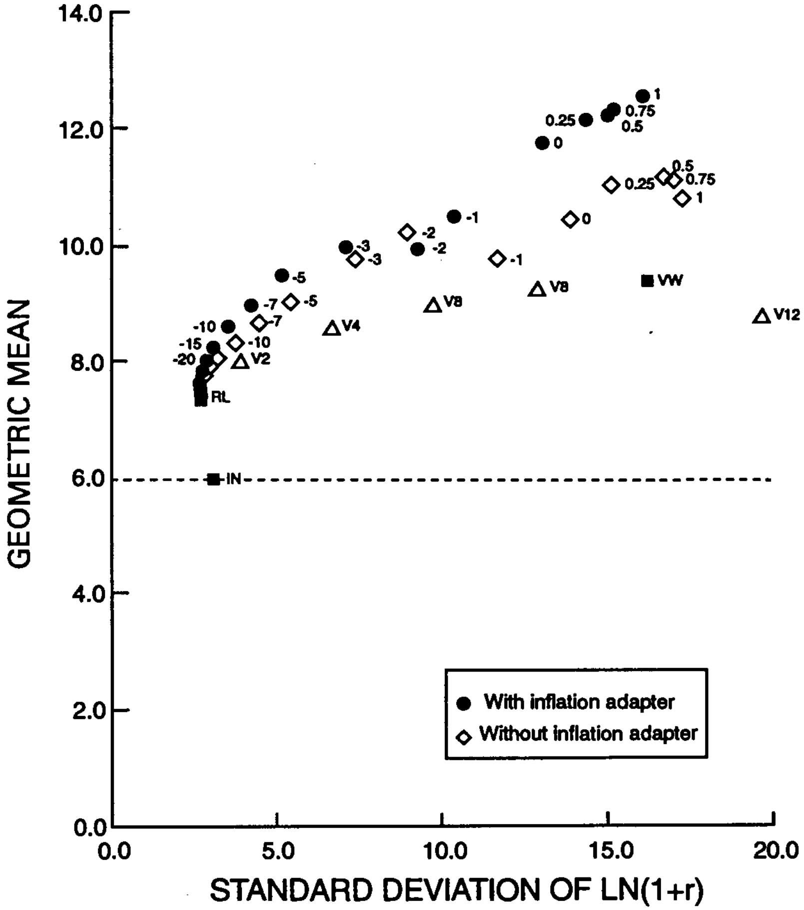

Figure 2. Geometric Means and Standard Deviations of Annual Returns for the Power Policies with and without the Inflation Adapter, Borrowing Permitted, 1966–1988 Treasury Bills and CRSP Value-Weighted Index (Quarterly portfolio revision, 32–quarter estimating period)

value-weighted index. Fifth, all the fixed-weight strategies that borrowed attained smaller geometric mean returns, with more variability, than the CRSP value-weighted index.

Table 2. Geometric Means and Standard Deviations of Annual Returns for sixteen Power Policies with and without the Inflation Adapter, 1966–1988 Treasury Bills and CRSP Value-Weighted Index (Quarterly portfolio revision, 32–quarter estimating period)

With Inflation Adapter			Without Inflation Adapter		
Portfolio	Geom. Mean	Std. Dev.	Portfolio	Geom. Mean	Std. Dev.
Panel A: Borrowing Precluded					
Power −75	7.59	2.61	Power −75	7.54	2.63
Power −50	7.68	2.63	Power −50	7.62	2.66
Power −30	7.85	2.71	Power −30	7.76	2.77
Power −20	8.06	2.86	Power −20	7.92	2.95
Power −15	8.26	3.04	Power −15	8.08	3.17
Power −10	8.60	3.50	Power −10	8.36	3.68
Power −7	8.98	4.17	Power −7	8.68	4.40
Power −5	9.48	5.13	Power −5	9.04	5.37
Power −3	10.33	6.69	Power −3	9.73	7.43
Power −2	10.83	7.74	Power −2	10.45	8.76
Power −1	11.00	8.10	Power −1	10.93	9.60
Power 0	11.25	8.58	Power 0	10.67	10.17
Power .25	11.24	8.71	Power .25	10.50	10.37
Power .5	11.22	8.79	Power .5	10.08	10.68
Power .75	11.32	8.75	Power .75	9.75	10.92
Power 1	11.30	8.74	Power 1	9.56	11.03
Panel B: Borrowing Permitted					
Power −3	9.99	6.98	Power −3	9.80	7.32
Power −2	9.96	9.27	Power −2	10.20	8.94
Power −1	10.51	10.38	Power −1	9.81	11.66
Power 0	11.78	13.07	Power 0	10.50	13.87
Power .25	12.17	14.39	Power .25	11.07	15.12
Power .5	12.26	15.00	Power .5	11.20	16.66
Power .75	12.36	15.15	Power .75	11.14	16.91
Power 1	12.59	16.02	Power 1	10.86	17.22
Panel C: Benchmarks					
RL	7.39	2.60	V12	8.80	19.62
V2	8.07	3.82	V14	8.03	23.18
V4	8.61	6.59	V16	7.09	26.86
V6	9.02	9.67	V18	5.98	30.72
V8	9.29	12.89	V20	4.68	34.77
VW	9.42	16.19	Inflation	5.96	3.04

6.2 The Investment Policies

Space does not permit a full analysis of the differences in investment policies. The following provides selected descriptive statistics for selected strategies with borrowing permitted, with and without the inflation adapter, over the 220 (92) quarters from 1934–88 (1966–88). Over the full period, the policies with (without) the inflation adapter were out of the market over 20 (between 13 and 16) percent of the time. But whenever funds were allocated to the market, the average commitment was greater with the inflation adapter. By way of contrast, during the 92 quarters from 1966–88, the policies with (without) the inflation adapter were out of the market approximately 49 (between 30 and 37) percent of the time. On the other hand, when funds were allocated to the market, the average commitment was again greater when the inflation adapter was in use than when it was not.

Table 3 provides more detail on the investment policies and rates of return of the logarithmic utility strategy, with and without the inflation adapter, over the 1966–88 period when borrowing is permitted. The inflation adapter caused this investor to behave much more conservatively from mid-1966 through 1970 for example – and at other times more aggressively.

7 Statistical Tests

There are a number of commonly accepted procedures for testing for abnormal investment performance:

(i) Jensen's (1968) test of selectivity, or microforecasting;

(ii) Merton's (1981), Henriksson and Merton's (1981) and Treynor and Mazuy's (1966) tests of market timing, or macroforecasting; and

(iii) a paired t-test of the difference in investment returns.

The first three of these tests embody both statistical and economic assumptions about the way assets are priced.

To compute the Jensen measure for portfolio j, we ran the regression

$$r_{jt} - r_{Lt} = \alpha_j + \beta_j(r_{mt} - r_{Lt}) + e_{jt}, \qquad (10)$$

where r_{mt} is taken to be the CRSP value-weighted index and r_{Lt} is the return on three-month Treasury bills. The intercept α_j is the measure of investment performance[12]. Positive (negative) values indicate superior (inferior) perfor-

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[12] While the Jensen measure is usually associated with the ability of a portfolio manager to select under-priced securities, a positive alpha may signify successful market timing if there is a quadratic relation between excess returns on the portfolio and the market as postulated in the Treynor–Mazuy test of market timing (see, for example, Sharpe and Alexander (1990) p. 755).

Table 3. Investment Policies and Returns of a Logarithmic Investor with and without the Inflation Adapter, Borrowing Permitted, 1966–1988 (Quarterly portfolio revision, 32–quarter estimating period)

With Inflation Adapter					Without Inflation Adapter				
Date	Return	Lend	Borrow	VW	Date	Return	Lend	Borrow	VW
1966-1	−3.75		−0.43	1.43	1966-1	−3.75		−0.43	1.43
1966-2	−6.33	1.00	−0.43	1.43	1966-2	−6.33		−0.43	1.43
1966-3	1.21	1.00			1966-3	−13.89		−0.43	1.43
1966-4	1.35	1.00			1966-4	7.91		−0.22	1.22
1967-1	1.19	1.00			1967-1	14.40		−0.02	1.02
1967-2	2.23			1.00	1967-2	2.49		−0.43	1.43
1967-3	3.28	0.67		0.33	1967-3	10.24		−0.43	1.43
1967-4	0.98			1.00	1967-4	0.68		−0.43	1.43
1968-1	1.27	1.00			1968-1	−9.59		−0.43	1.43
1968-2	1.34	1.00			1968-2	17.54		−0.43	1.43
1968-3	1.35	1.00			1968-3	4.04		−0.25	1.25
1968-4	1.70	0.76		0.24	1968-4	3.16		−0.25	1.25
1969-1	1.54	1.00			1969-1	−3.97		−0.25	1.25
1969-2	1.54	1.00			1969-2	−3.95			1.00
1969-3	1.75	1.00			1969-3	−3.78			1.00
1969-4	1.76	1.00			1969-4	0.67	0.21		0.79
1970-1	1.98	1.00			1970-1	1.98	1.00		
1970-2	1.62	1.00			1970-2	−11.93	0.37		0.63
1970-3	1.62	1.00			1970-3	13.85	0.24		0.75
1970-4	1.48	1.00			1970-4	10.00			1.00
1971-1	16.30		−0.54	1.54	1971-1	11.20			1.00
1971-2	−0.81		−0.54	1.54	1971-2	−0.81		−0.54	1.54
1971-3	1.00	0.82		0.18	1971-3	−0.77		−008	1.08
1971-4	6.42		−0.54	1.54	1971-4	4.75			1.00
1972-1	10.64		−0.82	1.82	1972-1	7.96		−0.29	1.29
1972-2	−1.00		−0.82	1.82	1972-2	−0.29		−0.30	1.30
1972-3	4.03		−0.82	1.82	1972-3	2.92			1.00
1972-4	10.23		−0.55	1.55	1972-4	7.21			1.00
1973-1	−3.68	0.39		0.61	1973-1	−6.95			1.00
1973-2	1.57	1.00			1973-2	−3.44	0.43		0.57
1973-3	2.00	1.00			1973-3	2.00	1.00		
1973-4	1.79	1.00			1973-4	1.79	1.00		
1974-1	1.94	1.00			1974-1	1.94	1.00		
1974-2	2.06	1.00			1974-2	2.06	1.00		
1974-3	1.94	1.00			1974-3	1.94	1.00		
1974-4	1.81	1.00			1974-4	1.81	1.00		
1975-1	1.62	1.00			1975-1	1.62	1.00		
1975-2	1.42	1.00			1975-2	1.42	1.00		
1975-3	1.54	1.00			1975-3	1.54	1.00		
1975-4	1.52	1.00			1975-4	1.52	1.00		
1976-1	15.16	0.06		0.94	1976-1	1.24	1.00		
1976-2	2.68		−0.03	1.03	1976-2	1.90	0.53		0.47
1976-3	1.83	0.09		0.91	1976-3	1.37	0.90		0.10
1976-4	4.08			1.00	1976-4	1.60	0.87		0.13

Table 3 (cont.).

With Inflation Adapter					Without Inflation Adapter				
Date	Return	Lend	Borrow	VW	Date	Return	Lend	Borrow	VW
1977-1	−12.22		−0.69	1.69	1977-1	−1.27	0.69		0.31
1977-2	5.35		−0.44	1.44	1977-2	1.58	0.86		0.14
1977-3	−2.91			1.00	1977-3	0.07	0.72		0.28
1977-4	1.47	1.00			1977-4	1.40	0.91		0.09
1978-1	1.52	1.00			1978-1	1.23	0.95		0.05
1978-2	1.61	1.00			1978-2	1.61	1.00		
1978-3	1.75	1.00			1978-3	6.89	0.29		0.71
1978-4	1.99	1.00			1978-4	0.49	0.81		0.19
1979-1	2.43	1.00			1979-1	2.43	1.00		
1979-2	2.37	1.00			1979-2	2.37	1.00		
1979-3	2.15	1.00			1979-3	2.15	1.00		
1979-4	2.52	1.00			1979-4	2.52	1.00		
1980-1	2.96	1.00			1980-1	2.96	1.00		
1980-2	3.68	1.00			1980-2	3.68	1.00		
1980-3	1.97	1.00			1980-3	1.97	1.00		
1980-4	2.85	1.00			1980-4	2.85	1.00		
1981-1	3.73	1.00			1981-1	3.73	1.00		
1981-2	3.24	1.00			1981-2	3.24	1.00		
1981-3	3.73	1.00			1981-3	3.73	1.00		
1981-4	3.77	1.00			1981-4	3.77	1.00		
1982-1	2.86	1.00			1982-1	2.86	1.00		
1982-2	3.46	1.00			1982-2	3.46	1.00		
1982-3	3.33	1.00			1982-3	3.33	1.00		
1982-4	22.71		−0.25	1.25	1982-4	22.29		−0.23	1.23
1983-1	12.91		−0.37	1.37	1983-1	10.55		−0.05	1.05
1983-2	18.04		−0.66	1.66	1983-2	17.63		−0.62	1.62
1983-3	−0.72		−0.25	1.25	1983-3	−1.07		−0.37	1.37
1983-4	−3.06		−1.00	2.00	1983-4	−3.06		−1.00	2.00
1984-1	−5.51		−0.44	1.44	1984-1	−5.32		−0.41	1.41
1984-2	−2.41			1.00	1984-2	−2.41			1.00
1984-3	9.63			1.00	1984-3	9.63			1.00
1984-4	1.97	0.11		0.89	1984-4	1.89			1.00
1985-1	13.09		−0.49	1.49	1985-1	10.89		−0.18	1.18
1985-2	12.30		−1.00	2.00	1985-2	12.30		−1.00	2.00
1985-3	−11.36		−1.00	2.00	1985-3	−11.36		−1.00	2.00
1985-4	30.94		−1.00	2.00	1985-4	30.94		−1.00	2.00
1986-1	25.42		−1.00	2.00	1986-1	25.42		−1.00	2.00
1986-2	8.53		−1.00	2.00	1986-2	8.53		−1.00	2.00
1986-3	−15.30		−1.00	2.00	1986-3	−15.30		−1.00	2.00
1986-4	6.44		−1.00	2.00	1986-4	6.44		−1.00	2.00
1987-1	36.80		−1.00	2.00	1987-1	36.80		−1.00	2.00
1987-2	5.77		−1.00	2.00	1987-2	5.77		−1.00	2.00
1987-3	10.35		−1.00	2.00	1987-3	10.35		−1.00	2.00
1987-4	−46.97		−1.00	2.00	1987-4	−46.97		−1.00	2.00
1988-1	11.90		−1.00	2.00	1988-1	10.94		−0.80	1.80
1988-2	10.68		−1.00	2.00	1988-2	10.68		−1.00	2.00
1988-3	−1.69		−1.00	2.00	1988-3	−1.62		−0.97	1.97
1988-4	2.91		−1.00	2.00	1988-4	2.83		−0.48	1.48

mance. The null hypothesis is that there is no superior investment performance and the alternative hypothesis is that there is. Thus, we report results of one-tailed tests for the Jensen test, as well as for the market timing and the paired t-test of differences in investment returns[13].

Next we considered the Henriksson–Merton test for market timing

$$r_{jt} - r_{Lt} = \alpha_j + \beta_{1j}(r_{mt} - r_{Lt}) + \beta_{2j} y_t + e_{jt}, \tag{11}$$

where $y_t = \max(0, r_{Lt} - r_{mt})$ may be interpreted as the payoff associated with a put option on the market portfolio with exercise price r_{Lt}. In this test it is assumed, in essence, that the investor places funds in equities when he expects an up-market and removes them when he expects a down-market. We may interpret α_j as a measure of microforecasting, β_{1j} as the up-market beta, and β_{2j} as the difference between the up and down-market betas. The null hypothesis of no timing ability is that $\beta_{2j} = 0$. We ran the regression as given as well as corrected for heteroscedasticity, using both White's (1980) correction and the correction suggested by Henriksson and Merton.

We also employed the Treynor–Mazuy test of market timing ability based on the regression

$$r_{jt} - r_{Lt} = \alpha_j + \beta_{1j}(r_{mt} - r_{Lt}) + \beta_{2j}(r_{mt} - r_{Lt})^2 + e_{jt}. \tag{12}$$

The null hypothesis of no timing ability is $\beta_{2j} = 0$. We again ran the regression both uncorrected as well as corrected for heteroscedasticity using White's (1980) correction. Finally, we turn to the paired t-test of differences in investment returns. Recall that terminal wealth w_0 in terms of beginning wealth w_n is given by

$$w_0 = w_n(1 + r_n)(1 + r_{n-1}) \ldots (1 + r_1) = w_n \exp\left[\sum_{t=1}^{n} \ln(1 + r_t)\right].$$

Since returns compound multiplicatively, we employ the paired t-test for dependent observations to the quarterly (and additive) variables $\ln(1 + r_t)$. Thus, to compare the return series $r_1^1, \ldots, r_n^1$ with the return series $r_1^2, \ldots, r_n^2$ for two different strategies, we calculate the statistic

$$t = \frac{\bar{d}}{\sigma(d)/\sqrt{n}}$$

where

$$\bar{d} = \sum_{t=1}^{n} \frac{\ln(1 + r_t^1) - \ln(1 + r_t^2)}{n}$$

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[13] The Jensen measure is not without its critics - see, for example, Roll (1977, 1978), Dybvig and Ross (1985), Green (1986), and Grauer (1991). While a number of concerns have been expressed, perhaps the most important is a possible reversal in rankings when different proxies are used for the market portfolio.

and $\sigma(d)$ is the standard deviation of $\ln(1 + r_t^1) - \ln(1 + r_t^2)$ null hypothesis is

$$\mathrm{E}\left[\ln\left(1 + r_t^1\right)\right] = \mathrm{E}\left[\ln\left(1 + r_t^2\right)\right]$$

while the alternative hypothesis is that

$$\mathrm{E}\left[\ln\left(1 + r_t^1\right)\right] > \mathrm{E}\left[\ln\left(1 + r_t^2\right)\right].$$

8 Test Results

8.1 The Jensen Tests

Table 4 shows the results when Jensen's measure of performance is applied to the power strategies with the inflation adapter for the period 1934–88. The results when borrowing is precluded are reported in Panel A, and the results with borrowing permitted in Panel B. Figure 3 presents the corresponding average excess return-beta plot.

It should be noted at the outset that performance has rarely been measured over periods of this length (55 years in this case). The previous example that comes to mind is the Black, Jensen, and Scholes (1972) test of the capital asset pricing model. In that paper, the authors measured the performance of ten beta-ranked portfolios over the 1931–65 period and found that low (high) beta portfolios earned positive (negative) abnormal returns. These abnormal returns were small, however, and with three exceptions statistically insignificant.

In light of this and other findings, the results for the power strategies with the inflation adapter are surprising. First, as Table 4 shows, all 25 alphas are positive; all are statistically significant at the 5 percent level, and eleven are statistically significant at the 1 percent level. Note that many of the alphas run about 40 percent of total excess returns. Second, with some minor exceptions for the less risk-averse strategies, the larger the beta of a strategy, the larger its abnormal return. Note that beta is greater than one for only two strategies when borrowing is permitted. When borrowing is precluded, the largest beta among the risk-averse strategies is .667.

Next we turn to the 1966–88 sub-period, which, like the full period, spans the 'crash' of October 1987. Since outliers can have significant effects in small samples, we report the results of all statistical tests in this period both with the fourth quarter of 1987 included in, as well as excluded from, the sample. Table 5 shows the results for the power strategies with the inflation adapter, both with borrowing precluded (Panel A) and with borrowing permitted (Panel B). Several observations stand out. First, both the average excess returns and the betas are uniformly smaller than for the full 1934–88

Table 4. Results from Applying the Jensen Performance Test

$$r_{jt} - r_{Lt} = \alpha_j + \beta_j(r_{mt} - r_{Lt}) + e_{jt}$$

to Quarterly Portfolio Returns obtained with sixteen Power Policies using the Inflation Adapter, 1934–1988 Treasury Bills and CRSP Value-Weighted Index (Quarterly portfolio revision, 32–quarter estimating period)

Portfolio		Excess Return*	Alpha	Prob. Alpha = 0	Beta	R^2
Panel A: Borrowing Precluded						
Power	−75	0.116	0.048	0.048	0.032	0.32
Power	−50	0.172	0.072	0.048	0.048	0.32
Power	−30	0.284	0.118	0.049	0.079	0.32
Power	−20	0.419	0.173	0.049	0.117	0.31
Power	−15	0.552	0.231	0.046	0.153	0.32
Power	−10	0.775	0.328	0.035	0.213	0.34
Power	−7	0.959	0.380	0.043	0.276	0.36
Power	−5	1.163	0.486	0.024	0.323	0.39
Power	−3	1.422	0.628	0.008	0.379	0.44
Power	−2	1.601	0.736	0.003	0.413	0.47
Power	−1	1.762	0.805	0.002	0.456	0.51
Power	0	1.962	0.841	0.001	0.535	0.59
Power	.25	2.002	0.819	0.001	0.565	0.62
Power	.5	2.085	0.802	0.002	0.612	0.65
Power	.75	2.098	0.699	0.006	0.667	0.68
Power	1	2.210	0.700	0.005	0.721	0.72
Panel B: Borrowing Permitted						
Power	−5	1.304	0.571	0.021	0.349	0.36
Power	−3	1.681	0.695	0.027	0.470	0.39
Power	−2	1.953	0.796	0.026	0.552	0.40
Power	−1	2.332	1.009	0.010	0.631	0.44
Power	0	2.879	1.265	0.003	0.770	0.51
Power	.25	3.044	1.276	0.003	0.843	0.54
Power	.5	3.233	1.233	0.007	0.954	0.57
Power	.75	3.531	1.206	0.012	1.109	0.62
Power	1	3.826	1.016	0.038	1.341	0.67

* (Average) excess return is measured in units of percent per quarter. The excess return on the CRSP value-weighted index was 2.10% per quarter.

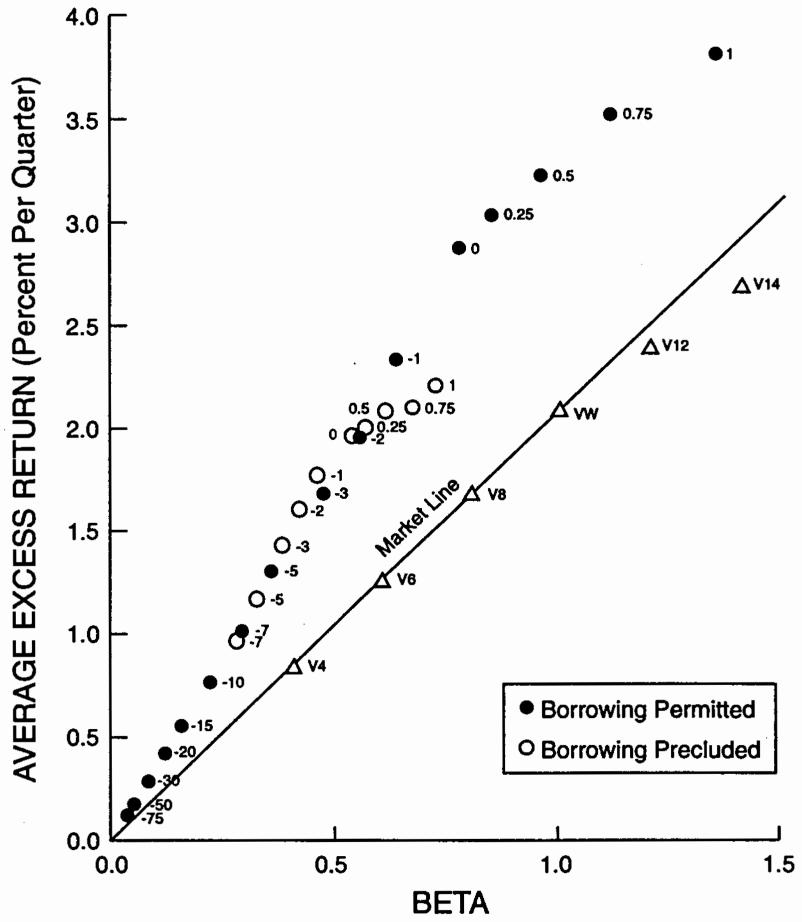

Figure 3. Average Quarterly Excess Returns and Betas for the Power Policies with the Inflation Adapter, 1934–1988 Treasury Bills and CRSP Value-Weighted Index (Quarterly portfolio revision, 32–quarter estimating period)

period. Second, when the fourth quarter of 1987 is excluded, the average excess returns and the abnormal returns are uniformly larger, and the betas are uniformly smaller, than when the fourth quarter is included. Third, all the alphas are positive. When the fourth quarter of 1987 is included, none of the alphas are statistically significant at the 5% level and their values are between 50 and 75 percent of the corresponding values when the fourth quarter is excluded. However, when the fourth quarter of 1987 is excluded, all of the alphas are statistically significant at the 2.5% level.

To place the above results in perspective, we summarize the results for 130 mutual funds for the 1968–82 sub-period and contrast them with the results for the active strategies when the investment universe consists of either cash and stocks, or of cash, long-term US government bonds, long-term US corporate bonds, the S&P 500 index, and an index of small stocks. The average excess return-beta plot for the 130 mutual funds and the power policies based on the stocks and cash universe is shown in Figure 4. The average excess return on the market was .32 percent per quarter. The betas of the mutual funds averaged .95, ranging from .36 to 1.64. Fifty-seven (73) of the funds had positive (negative) abnormal returns. Furthermore, only 13 (22) of the positive (negative) alphas were statistically significant at the 5% level. On the other hand, the betas of the power strategies using the inflation adapter averaged only .12, ranging from .005 to .297. All 24 strategies exhibited positive abnormal returns. Moreover, the alphas averaged .58 percent per quarter and 16 of them were statistically significant at the 5% level.

In separate runs, with the investment universe consisting of cash, long-term US government bonds, long-term US corporate bonds, the S&P 500 index, and an index of small stocks, the active strategies produced even larger excess and abnormal returns (averaging .71 percent per quarter) and only marginally higher betas over the same (1968–82) period.

8.2 The Henriksson–Merton and Treynor–Mazuy Tests

While many have tried to time the market, the empirical evidence on the extent of success is mixed. For example, the evidence presented in Chang and Lewellen (1984), Henriksson (1984), Kon (1983), Treynor and Mazuy (1966), and confirmed in our sample of mutual funds, indicates that a greater number of the funds studied have exhibited negative timing ability than have displayed positive timing success. We were therefore not surprised that, over the full 1934–88 period, we found little evidence of market timing ability by the power strategies as measured by either the Henriksson–Merton or the Treynor–Mazuy test.

By way of contrast, Table 6 shows the results for the Henriksson–Merton test of market timing ability when the inflation adapter was employed during the 1966–88 sub-period. With all quarters included, the tests showed that 16 (8) strategies exhibited positive (negative) timing ability, with the negative sign concentrated among the more risk-averse powers. None of the strategies displayed statistically significant timing ability (although the less risk-averse powers without margin were not far off).

However, with the (outlier) fourth quarter of 1987 excluded, the results

Table 5. Results from Applying the Jensen Performance Test

$$r_{jt} - r_{Lt} = \alpha_j + \beta_j(r_{mt} - r_{Lt}) + e_{jt}$$

to Quarterly Portfolio Returns obtained with sixteen Power Policies using the Inflation Adapter, 1966–1988 Treasury Bills and CRSP Value-Weighted Index (Quarterly portfolio revision, 32–quarter estimating period)

	Results including 1987Q4				Results Excluding 1987Q4			
Portfolio	Excess Return*	Alpha	Prob. Alpha=0	Beta	Excess Return	Alpha	Prob. Alpha=0	Beta
Panel A: Borrowing Precluded								
Power −75	0.046	0.027	0.210	0.022	0.071	0.053	0.022	0.016
Power −50	0.068	0.040	0.210	0.033	0.106	0.079	0.022	0.023
Power −30	0.113	0.065	0.210	0.055	0.175	0.131	0.022	0.039
Power −20	0.166	0.096	0.211	0.081	0.258	0.193	0.022	0.057
Power −15	0.218	0.126	0.211	0.107	0.339	0.253	0.022	0.075
Power −10	0.317	0.183	0.212	0.155	0.493	0.369	0.022	0.109
Power −7	0.436	0.251	0.213	0.214	0.679	0.508	0.022	0.150
Power −5	0.575	0.355	0.163	0.254	0.846	0.635	0.017	0.185
Power −3	0.792	0.534	0.090	0.297	1.064	0.801	0.011	0.231
Power −2	0.923	0.641	0.066	0.325	1.197	0.899	0.010	0.261
Power −1	0.969	0.674	0.060	0.340	1.244	0.928	0.010	0.277
Power 0	1.039	0.724	0.053	0.363	1.315	0.970	0.010	0.302
Power .25	1.038	0.719	0.056	0.367	1.313	0.963	0.011	0.307
Power .5	1.035	0.714	0.058	0.370	1.311	0.957	0.012	0.310
Power .75	1.060	0.737	0.053	0.373	1.336	0.979	0.011	0.312
Power 1	1.055	0.732	0.055	0.373	1.331	0.974	0.011	0.313
Panel B: Borrowing Permitted								
Power −3	0.798	0.476	0.184	0.371	1.198	0.893	0.020	0.267
Power −2	0.938	0.525	0.221	0.476	1.472	1.089	0.023	0.336
Power −1	1.115	0.656	0.184	0.528	1.662	1.218	0.022	0.388
Power 0	1.483	0.950	0.116	0.614	2.034	1.485	0.017	0.481
Power .25	1.611	1.047	0.103	0.650	2.164	1.569	0.017	0.521
Power .5	1.652	1.070	0.103	0.670	2.205	1.586	0.020	0.542
Power .75	1.678	1.094	0.099	0.673	2.232	1.609	0.018	0.545
Power 1	1.761	1.149	0.095	0.704	2.315	1.653	0.020	0.579

* (Average) excess return is measured in units of percent per quarter. In the 1966–1988 period the excess return on the CRSP value-weighted index was 0.87% per quarter including the fourth quarter of 1987 and 1.14% per quarter excluding the fourth quarter of 1987.

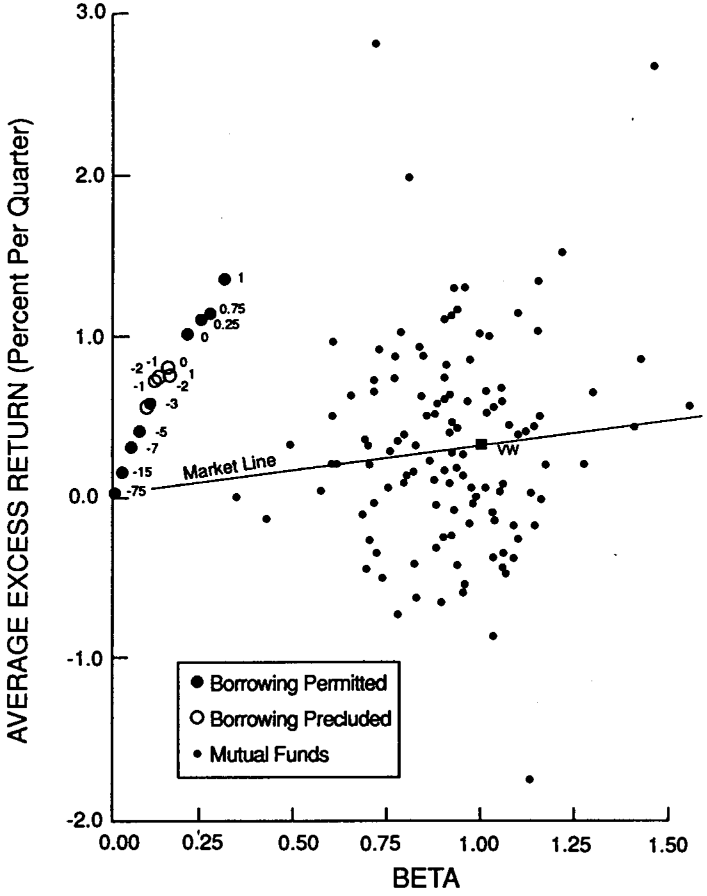

Figure 4. Average Quarterly Excess Returns and Betas for the Power Policies with the Inflation Adapter and for 130 Mutual Funds, 1968–1982 Treasury Bills and CRSP Value-Weighted Index (Quarterly portfolio revision, 32–quarter estimating period)

differ substantially. First, there are substantial differences between the up and down-market betas. The up-market betas are at least 1.7 times the the Jensen betas estimated in Table 5; none of the down-market betas exceeds .095. Second, some of the numbers now suggest timing ability of the statistically significant variety. All 24 strategies exhibited positive timing at the 1% level, both without correction as well as with the Henriksson–Merton correction for

Table 6. Summary of Results from Applying the Henriksson–Merton (HM) Market Timing Test

$$r_{jt} - r_{Lt} = \alpha_j + \beta_{1j}(r_{mt} - r_{Lt}) + \beta_{2j}y_t + e_{jt}$$

to Quarterly Portfolio Returns obtained with sixteen Power Policies using the Inflation Adapter, 1966–1988 Treasury Bills and CRSP Value-Weighted Index (Quarterly portfolio revision, 32–quarter estimating period)

	Results including 1987Q4					Results excluding 1987Q4				
Portfolio	Up-Market Beta β_1	Down-Market Beta $\beta_1 - \beta_2$	No Corr. Prob. $\beta_2 = 0$	White Corr. Prob. $\beta_2 = 0$	HM Corr. Prob. $\beta_2 = 0$	Up-Market Beta β_1	Down-Market Beta $\beta_1 - \beta_2$	No Corr. Prob. $\beta_2 = 0$	White Corr. Prob. $\beta_2 = 0$	HM Corr. Prob. $\beta_2 = 0$
Panel A: Borrowing Precluded										
Power −75	0.022	0.023	0.47	0.48	0.47	0.028	0.003	0.01	0.04	0.00
Power −50	0.033	0.034	0.47	0.49	0.47	0.041	0.004	0.01	0.04	0.00
Power −30	0.054	0.056	0.47	0.49	0.47	0.068	0.006	0.01	0.04	0.00
Power −20	0.079	0.083	0.47	0.49	0.47	0.101	0.009	0.01	0.04	0.00
Power −15	0.104	0.109	0.47	0.49	0.47	0.132	0.012	0.01	0.04	0.00
Power −10	0.152	0.158	0.47	0.49	0.46	0.193	0.018	0.01	0.04	0.00
Power −7	0.209	0.218	0.47	0.49	0.46	0.265	0.024	0.01	0.04	0.00
Power −5	0.267	0.242	0.43	0.47	0.41	0.329	0.026	0.00	0.03	0.00
Power −3	0.348	0.247	0.24	0.37	0.24	0.410	0.034	0.00	0.02	0.00
Power −2	0.401	0.251	0.17	0.31	0.17	0.462	0.040	0.00	0.02	0.00
Power −1	0.422	0.260	0.15	0.30	0.16	0.483	0.051	0.00	0.02	0.00
Power 0	0.467	0.263	0.11	0.26	0.11	0.527	0.056	0.00	0.01	0.00
Power .25	0.473	0.265	0.10	0.26	0.11	0.533	0.060	0.00	0.01	0.00
Power .5	0.475	0.269	0.11	0.26	0.12	0.535	0.065	0.00	0.01	0.00
Power .75	0.477	0.272	0.11	0.26	0.12	0.536	0.068	0.00	0.01	0.00
Power 1	0.478	0.271	0.11	0.26	0.12	0.537	0.067	0.00	0.01	0.00
Panel B: Borrowing Permitted										
Power −3	0.381	0.361	0.46	0.48	0.44	0.473	0.042	0.00	0.04	0.00
Power −2	0.476	0.476	0.50	0.50	0.41	0.600	0.047	0.00	0.03	0.00
Power −1	0.566	0.491	0.39	0.45	0.37	0.692	0.057	0.00	0.03	0.00
Power 0	0.722	0.509	0.23	0.36	0.23	0.846	0.082	0.00	0.02	0.00
Power .25	0.792	0.513	0.18	0.32	0.19	0.916	0.089	0.00	0.01	0.00
Power .5	0.834	0.511	0.15	0.30	0.16	0.957	0.087	0.00	0.01	0.00
Power .75	0.836	0.516	0.15	0.30	0.16	0.959	0.093	0.00	0.01	0.00
Power 1	0.915	0.501	0.10	0.26	0.10	1.038	0.078	0.00	0.01	0.00

The probability that $\beta_2 = 0$ is reported three ways: with no correction for heteroskedasticity, with the White correction for heteroskedasticity, and with the Henriksson–Merton correction for heteroskedasticity.

heteroscedasticity. Based on the White correction for heteroscedasticity, 24 (1) strategies displayed positive timing ability at the 5% (1%) level.

Table 7 summarizes the results of the Henriksson–Merton and Treynor–Mazuy market timing tests for the 1966–88 sub-period with and without correcting for heteroscedasticity. When the fourth quarter of 1987 is excluded, all 24 alphas in the HM tests are negative (though not significantly so). This is consistent with the well-known observation (see e.g. Henriksson (1984) and

Jagannathan and Korajczyk (1986)) that the microforecasting and macroforecasting measures are negatively correlated. This is troublesome in that the strategies under study here did not attempt to apply any microforecasting.

The Treynor-Mazuy results also lend support to market timing ability on the part of the 24 power strategies. When the fourth quarter of 1987 was included, 13 of the β_2-coefficients were positive. When the fourth quarter of 1987 was excluded, all 24 coefficients were statistically significant at the 1% percent level without correcting for heteroscedasticity, while 15 were statistically significant at the 5% level after White's correction for heteroscedasticity.

To add some perspective to the preceding analysis, we compare the results for 130 mutual funds over the 1968–82 period to the results for the active strategies when the investment universe consisted of either cash and stocks, or of cash, long-term US government bonds, long-term US corporate bonds, the S&P 500 index, and an index of small stocks. According to the Henriksson-Merton test, 60 (70) mutual funds displayed positive (negative) timing ability, but only 5 (11) of the associated return series were statistically significant at the 5% level. On the other hand, when the investment universe consisted of stocks and cash, all 24 power strategies exhibited positive timing ability at the 5% level. With the larger investment universe, 24 (21) of the power strategies displayed positive timing ability at the 5% (1%) level; remarkably, 19 of the 24 down-market betas were negative.

8.3 The Paired *t*-Tests

Recall that, with minor exceptions, the returns of the power strategies with the inflation adapter dominated the corresponding returns without the inflation adapter, both for the full 1934–88 and for the 1966–88 sub-period. However, based on the paired *t*-test, none of these differences were statistically significant.

The returns earned by the strategies using the inflation adapter were also compared with the returns of various other benchmarks. Results for selected pairs are shown in Table 8. Over the 1934–88 period, the very risk averse strategies −75, −50, and −30 generated higher returns than the risk-free asset at the 1% level of significance; the same is true for powers 0 and .25 when compared to fixed-weight strategy V6 but not when compared to policy V8. All other comparisons (except two) of active and fixed-weight strategies of similar risk yielded insignificant differences over the full 1934-88 period.

In the 1966–88 sub-period, statistical significance hinged to a high degree on the presence or absence of the (outlier) fourth quarter of 1987. When this quarter was *included*, none of the comparisons between the active and the fixed-weight strategies of similar risk was found to be statistically significant, as Table 8 shows. However, when the fourth quarter of 1987 was *excluded*,

Table 7. Summary of Results from Applying the Henriksson-Merton (HM) Market Timing Test

$$r_{jt} - r_{Lt} = \alpha_j + \beta_{1j}(r_{mt} - r_{Lt}) + \beta_{2j}y_t + e_{jt}$$

and the Treynor-Mazuy (TM) Market Timing Test

$$r_{jt} - r_{Lt} = \alpha_j + \beta_{1j}(r_{mt} - r_{Lt}) + \beta_{2j}(r_{mt} - r_{Lt})^2 + e_{jt}$$

to Quarterly Portfolio Returns obtained with sixteen Power Policies using the Inflation Adapter, 1966–1988 Treasury Bills and CRSP Value-Weighted Index (Quarterly portfolio revision, 32-quarter estimating period)

	Results including 1987Q4				Results excluding 1987Q4			
	α		β_2		α		β_2	
	Pos	Neg	Pos	Neg	Pos	Neg	Pos	Neg
	HM Test with No Correction for Heteroskedasticity							
count	19	5	16	8	0	24	24	0
significant at 5%	0	0	0	0	0	0	24	0
significant at 1%	0	0	0	0	0	0	24	0
	HM Test with White Correction for Heteroskedasticity							
count	19	5	16	8	0	24	24	0
significant at 5%	0	0	0	0	0	0	24	0
significant at 1%	0	0	0	0	0	0	1	0
	HM Test with HM Correction for Heteroskedasticity							
count	21	3	16	8	0	24	24	0
significant at 5%	0	0	0	0	0	0	24	0
significant at 1%	0	0	0	0	0	0	24	0
	TM Test with No Correction for Heteroskedasticity							
count	24	0	13	11	24	0	24	0
significant at 5%	0	0	0	0	0	0	24	0
significant at 1%	0	0	0	0	0	0	24	0
	TM Test with White Correction for Heteroskedasticity							
count	24	0	13	11	24	0	24	0
significant at 5%	0	0	0	0	0	0	15	0
significant at 1%	0	0	0	0	0	0	0	0

Table 8. Results from Applying the Paired t-test of Differences in Investment Returns to Quarterly Portfolio Returns obtained with Selected Power Policies using the Inflation Adapter, 1934–88 and 1966–88 Treasury Bills and CRSP Value-Weighted Index (Quarterly portfolio revision, 32–quarter estimating period)

	1934–1988			1966–1988			
				Including 87Q4		Excluding 87Q4	
Power versus Benchmark	$\bar{d}$	Prob. $\bar{d}=0$	Power versus Benchmark	$\bar{d}$	Prob. $\bar{d}=0$	$\bar{d}$	Prob. $\bar{d}=0$
Panel A: Borrowing Precluded							
−75 vs. RL	0.0011	0.000	−75 vs. RL	0.0004	0.122	0.0007	0.009
−50 vs. RL	0.0017	0.001	−50 vs. RL	0.0007	0.124	0.0010	0.009
−30 vs. RL	0.0027	0.001	−30 vs. RL	0.0011	0.128	0.0017	0.009
			−20 vs. RL	0.0016	0.134	0.0025	0.009
−20 vs. V2	−0.0000	0.492					
−15 vs. V2	0.0012	0.187	−15 vs. V2	0.0004	0.403	0.0011	0.244
			−10 vs. V2	0.0012	0.296	0.0026	0.091
−10 vs. V4	−0.0005	0.396	−7 vs. V2	0.0021	0.255	0.0042	0.040
−7 vs. V4	0.0010	0.333	−5 vs. V2	0.0033	0.189	0.0057	0.020
−5 vs. V6	−0.0006	0.418	−3 vs. V4	0.0039	0.167	0.0058	0.054
−3 vs. V6	0.0017	0.276	−2 vs. V4	0.0051	0.117	0.0070	0.034
−2 vs. V6	0.0033	0.118	−1 vs. V4	0.0054	0.103	0.0074	0.029
1 vs. V6	0.0047	0.041	0 vs. V4	0.0060	0.086	0.0079	0.024
0 vs. V6	0.0064	0.008	.25 vs. V4	0.0060	0.089	0.0079	0.025
.25 vs. V6	0.0067	0.005	.5 vs. V4	0.0059	0.092	0.0079	0.027
			.75 vs. V4	0.0062	0.084	0.0081	0.024
0 vs. V8	0.0033	0.137	1 vs. V4	0.0061	0.086	0.0081	0.025
.25 vs. V8	0.0036	0.110					
.5 vs. V8	0.0042	0.068	.75 vs. V6	0.0052	0.146	0.0066	0.087
			1 vs. V6	0.0052	0.148	0.0065	0.088
.75 vs. VW	0.0024	0.253					
1 vs. VW	0.0034	0.161	1 vs. V8	0.0136	0.081	0.0053	0.192
Panel B: Borrowing Permitted							
−5 vs. V6	0.0005	0.432	−3 vs. V4	0.0032	0.284	0.0068	0.052
			−2 vs. V4	0.0031	0.344	0.0090	0.036
−3 vs. V8	0.0002	0.480					
−2 vs. V8	0.0021	0.317	−1 vs. V6	0.0034	0.337	0.0089	0.066
			0 vs. V6	0.0063	0.230	0.0118	0.035
−1 vs. VW	0.0025	0.303					
0 vs. VW	0.0069	0.069	0 vs. V8	0.0056	0.256	0.0106	0.070
.25 vs. VW	0.0079	0.046	.25 vs. V8	0.0065	0.230	0.0114	0.061
			.5 vs. V8	0.0067	0.226	0.0116	0.061
0 vs. V12	0.0056	0.137	.75 vs. V8	0.0069	0.218	0.0119	0.058
.25 vs. V12	0.0066	0.095	1 vs. V8	0.0074	0.206	0.0124	0.055
.5 vs. V12	0.0074	0.074					
			.75 vs. VW	0.0066	0.238	0.0109	0.097
.75 vs. V14	0.0078	0.080	1 vs. VW	0.0072	0.224	0.0114	0.090
1 vs. V16	0.0074	0.113					

many differences were statistically significant, especially in the no borrowing case. Over this sub-period, the very risk averse strategies again outdistanced the risk-free asset at the 1% level. In addition, many of the rather risk-tolerant policies outperformed various comparable fixed-weight portfolios at the 5% level of significance; this was not true for the most risk-tolerant strategies, however.

9 Concluding Remarks

Several conclusions emerge. First, the results suggest that adding a simple inflation adapter to the empirical probability assessment approach, which uses only the past to (naively) forecast the future, provides an improvement in applications of discrete-time dynamic investment theory. This improvement was especially notable in the sub-periods beginning in 1966, when inflation changes were most significant, especially when the fourth quarter of 1987 was excluded from the sample.

Second, the simple inflation adapter used appears to be biased toward conservatism (see Figures 1, 2 and 4). Recall that when probability estimates are unbiased, the logarithmic strategy (power 0) should asymptotically have the highest geometric mean.

Third, the empirical probability assessment approach employing the inflation adapter performed well when compared to the up and down-levered value-weighted market portfolio, with several statistically significant excess returns to its credit. The model had only two assets at its disposal with which to implement timing: cash and the value-weighted market.

Fourth, the various standard portfolio performance measures provided somewhat contradictory results. The Jensen test rated the model's performance very high over the full period, but rated it lower for the 1966–88 sub-period. The Henriksson–Merton and the Treynor–Mazuy tests, on the other hand, saw the full period as quite average, but gave a highly favorable report on the 1966–88 sub-period when the fourth quarter of 1987 was excluded. The paired t-test weighed in somewhere in between.

The reader should also be reminded of the limitations of the study. The model used focuses on sequential reinvestment only, without concern for intermediate consumption; even though its birth occurred in the mid-seventies, it was applied as far back as 1934. The latter statement also applies at least partially to the data base used. The joint probability estimates were based, on a moving basis, on the most recent eight years only[14]. All investors were assumed to be strict price-takers. Transactions costs and taxes were ignored (as in the underlying returns series); turnover, however, was

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[14] Use of a ten-year estimating period produced similar results.

low (see e.g. Table 3). Finally, maintenance margins were ignored whenever leverage was used. Nevertheless, the simple inflation adapter used in this study has substantial power. One potential explanation is that the empirical assessment approach, aided by the inflation adapter, is able to exploit the kind of intermediate to long-term mean reversion documented by e.g. Fama and French (1988a,b) and Poterba and Summers (1988) – and to capture some of the fruits of technical analysis reported in Brock, Lakonishok, and LeBaron (1992).

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